FE implementation of a 3D non-linear solids solver

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Content:

Aims:

- Interface with polymer melt FE work
- Gain knowledge of energy formulations / solids techniques
- Broaden horizons

Method:

Cauchy's Equation of Motion:

$$\nabla\cdot\boldsymbol{\sigma}+\boldsymbol{F}=\rho\,\ddot{\boldsymbol{u}}$$,

or in Cartesian co-ordinates

$$\frac{\partial}{\partial x}(\sigma_{xx})+\frac{\partial}{\partial y}(\sigma_{xy})+\frac{\partial}{\partial z}(\sigma_{xz})+F_x=\rho\frac{d^2u}{dt^2},$$
$$\frac{\partial}{\partial x}(\sigma_{xy})+\frac{\partial}{\partial y}(\sigma_{yy})+\frac{\partial}{\partial z}(\sigma_{yz})+F_y=\rho\frac{d^2v}{dt^2},$$
$$\frac{\partial}{\partial x}(\sigma_{xz})+\frac{\partial}{\partial y}(\sigma_{yz})+\frac{\partial}{\partial z}(\sigma_{zz})+F_z=\rho\frac{d^2w}{dt^2},$$.

noting $|\sigma|=|\tau|-\delta_{ij}\,p,$ that

i.e.

$$\sigma_{xx}=\tau_{xx}-p,$$
$$\sigma_{yy}=\tau_{yy}-p,$$
$$\sigma_{zz}=\tau_{zz}-p,$$
$$\tau_{xy}=\sigma_{xy},$$
$$\tau_{xz}=\sigma_{xz},$$
$$\tau_{yz}=\sigma_{yz},$$

Fluids

For Newtonian flows:

$$\boldsymbol{\tau}_{ij}=\mu\left(\frac{\partial \dot{u}_i}{\partial x_j}+\frac{\partial \dot{u}_j}{\partial x_i}\right)$$

eg.

$$\tau_{xx}=2\mu\frac{\partial \dot{u}}{\partial x},$$

$$\tau_{xy}=\mu\left(\frac{\partial \dot{u}}{\partial y}+\frac{\partial \dot{v}}{\partial x}\right),$$

Solids

For *small strain solids* (using penalty method)

$$\boldsymbol{\sigma}=[\boldsymbol{D}]\boldsymbol{e}$$

where:

$$[\boldsymbol{D}]=G\begin{bmatrix} \frac{4}{3} & -\frac{2}{3} & -\frac{2}{3} & 0 & 0 & 0 \\ -\frac{2}{3} & \frac{4}{3} & -\frac{2}{3} & 0 & 0 & 0 \\ -\frac{2}{3} & -\frac{2}{3} & \frac{4}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}+\frac{1}{\epsilon}\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

where: $G=\frac{E}{2(1+\nu)}$.

Non-linear solids:

$$\sigma \text{ from } U = f(I_{1,} I_{2,} I_3)$$

e.g.

$$U = \frac{G}{2}(I_1 - 3)$$

FE formulation of:

$$\boldsymbol{\tau} = \boldsymbol{\tau}^E + \boldsymbol{\tau}^\mu$$

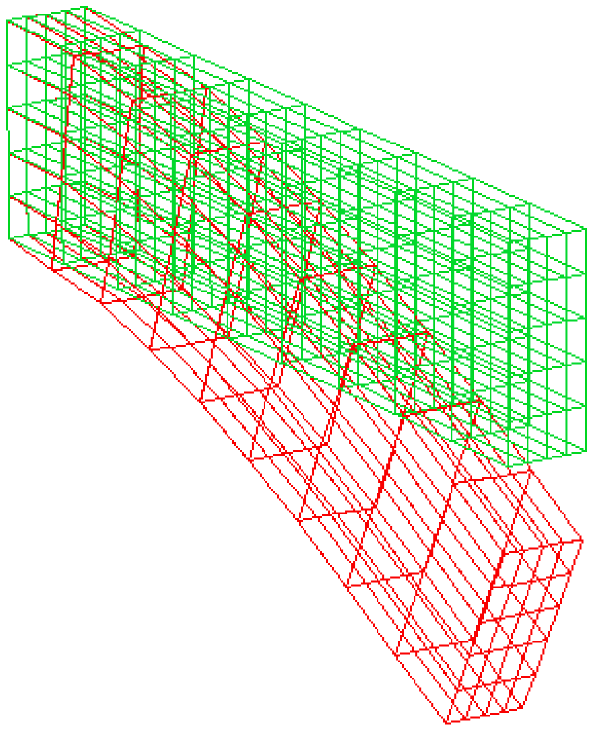
gives matrix equation:

$$[M]\ddot{\boldsymbol{u}} + [C]\dot{\boldsymbol{u}} + [K]\boldsymbol{u} = \boldsymbol{F}$$

-> Newmark time integration scheme

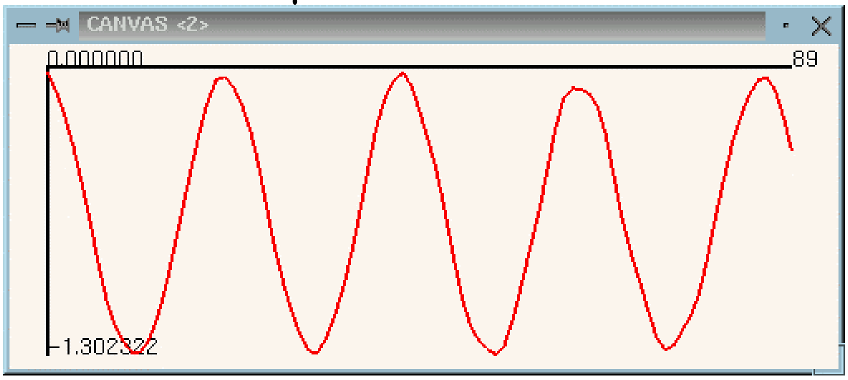
Initial Tests:

Beam with force suddenly applied to end:

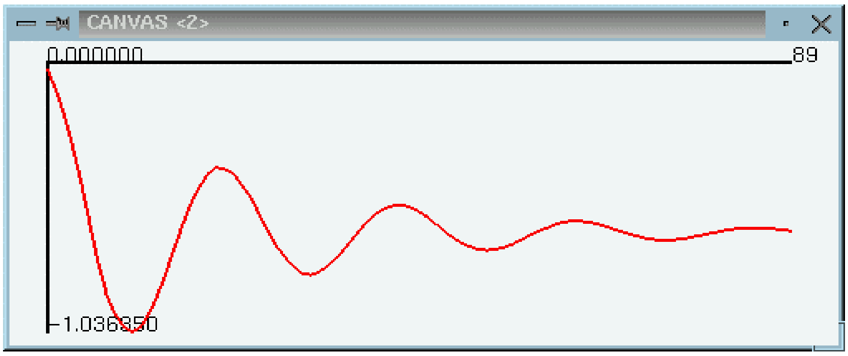


Using $U = \frac{G}{2}(I_1 - 3)$ **, G=2e6, end node displacement**

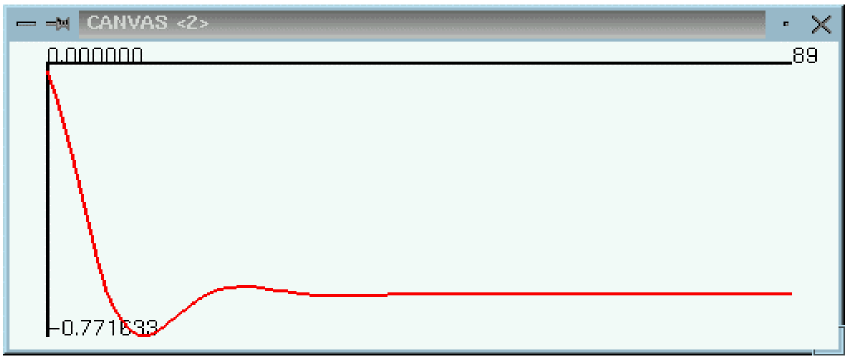
μ=0.0 :



μ=1.0 :



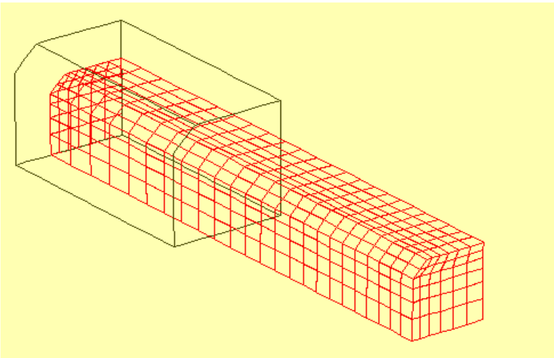
μ=4.0 :



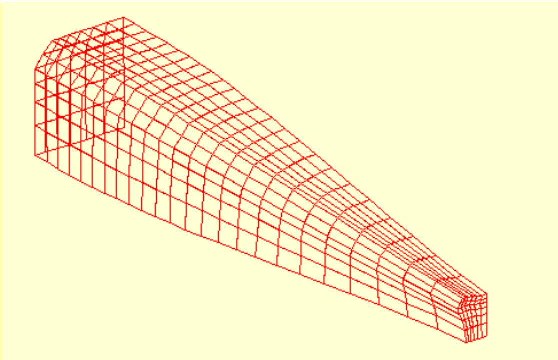
-> time

Hyper-elastic problems (with J. Sweeney)

Ball Model (attempt)

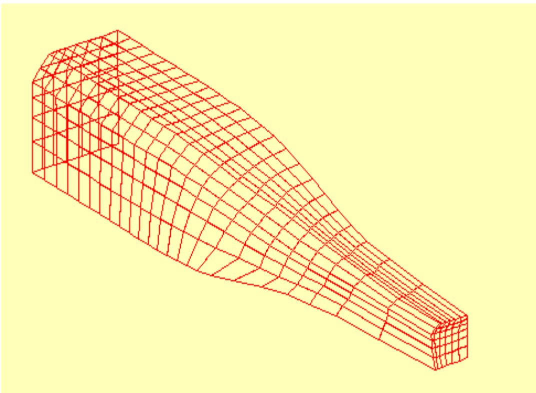


stretched ..

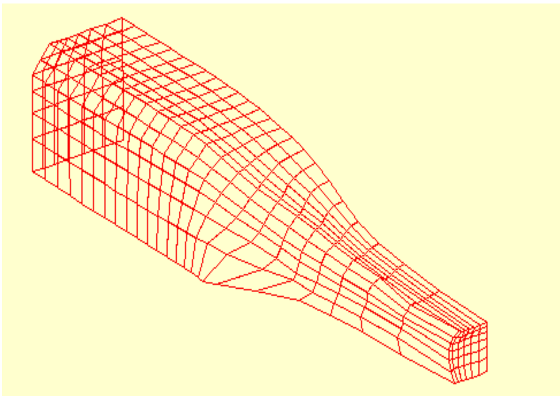


.. starting ..

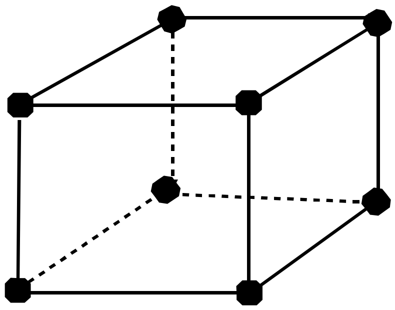
.. promising ......



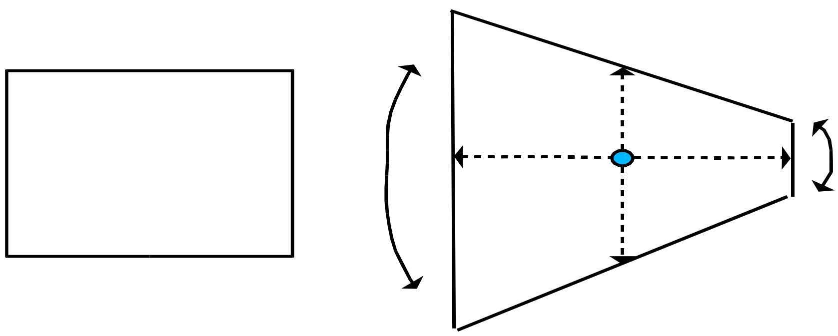
.. not again!



Deformation Problems - 8-node linear 'brick'



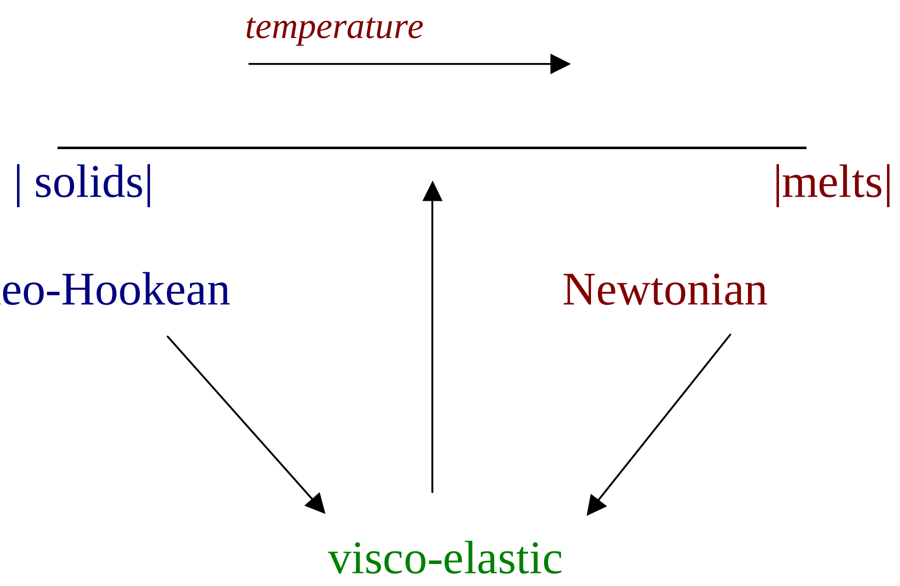
in 2D



-> quadratic elements

Melt-Solid interface:

suitable constitutive equations:



suitable solution parameters: