DISPERSION OF POLLUTANTS DISCHARGED INTO THE OCEAN: THE INTERACTION OF SMALL AND LARGE SCALE PHENOMENA

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In many coastal towns, sewage and industrial effluents are discharged via coastal outfalls into the sea [3,9]. Here we study the discharge of an axisymmetric buoyant jet of less salty and colder wastewater into an unstratified stagnant sea to form a cloud of pollutant at the sea surface. The transport of pollutants arising from double diffusion at the lower interface of this cloud and the re-entrainment of surrounding seawater into the turbulent rising jet are taken into account. These have not been considered in conventional models hitherto [2]. We model the evolution of the temperature, salt and pollutant concentrations in the buoyant jet, polluted cloud and in the seawater surrounding the jet. The conservation equations are solved numerically. Two outfall scenarios corresponding to low and high interface stability are considered. We find that the interaction between double diffusion and turbulent entrainment into the buoyant jet results in a local accumulation of pollutant. When the interface stability is low, the pollutant concentration near the buoyant jet can increase by 100% compared to the magnitude expected in the absence of double-diffusion, in the timescale required for the polluted cloud to reach the coast. The effect of double diffusion on pollutant transport in the high stability case is found to be small, as expected.

Double-diffusive convection, outfall discharge

INTRODUCTION

In many coastal towns, sewage and industrial effluents are usually discharged via coastal outfalls into the sea [3,9]. Wastewater is typically less dense than seawater and hence rises as a buoyant jet above the outfall. In an unstratified sea, it then spreads out at the sea surface as a less salty and colder cloud of pollutant above the warmer and saltier seawater. Dilution of the polluted cloud thereafter is thought to be mainly controlled by horizontal turbulent and molecular diffusion [2,3]. Vertical transport is usually considered to be small, as diffusion effects are orders of magnitude smaller than advective horizontal transport.

However, the temperature and salt concentration of the polluted cloud have opposing effects on the vertical density. As a result an unstable boundary layer may grow at the interface between the polluted cloud and the underlying seawater. Break-up of this boundary layer can drive vigorous buoyant convection above and below the interface. This convective phenomenon is caused by the presence of two components (heat and salt) with different molecular diffusivities and is known as double diffusion [13]. With the colder and less salty cloud overlaying the warmer and saltier seawater, a *diffusive type* interface is formed. Laboratory experiments [4,6,7,12] have shown that the double diffusive fluxes across such an interface can be much larger than the vertical transport in a single component fluid.

Double diffusion develops on length-scales of the order of 1 mm in the ocean. However, current computational fluid dynamic simulations for submerged discharges in the ocean consider length-scales of the order of ten to hundreds of metres [2]. The resolution of these models cannot therefore capture the effect of double-diffusion. In this work, we develop a theoretical model that takes into account the effect of small-scale phenomena on the large-scale transport and mixing of the pollutant.

In this paper, we consider the discharge of an axisymmetric buoyant jet of less salty and colder wastewater into an unstratified stagnant sea to form a cloud of pollutant at the sea surface. The transport of pollutants arising from double diffusion at the lower interface of this cloud and the re-entrainment of surrounding seawater into the turbulent rising jet are taken into account. Sea currents are assumed to have a negligible effect. This is an important worst case scenario as currents usually increase dilution [11].

We conduct new laboratory experiments to investigate the motion in the jet, in the polluted cloud and in the surrounding seawater. Our experimental observations are used in the development of our theoretical model. We model the evolution of the temperature, salt and pollutant concentrations in the buoyant jet, polluted cloud and in the seawater surrounding the jet. The conservation equations are solved numerically. Two outfall scenarios corresponding to low and high interface stability are presented to assess the impact of double diffusion on discharges from coastal outfalls.

EXPERIMENTS

The experiments were conducted in a horizontal Perspex channel of width 25 cm and length 9.7 m. The channel was initially filled with water to a depth of approximately 28 cm and temperature 22 °C. A two-dimensional, line jet was generated at the upstream wall of the channel. The jet fluid consisted of an aqueous solution of NaCl with density 1.0009 g/cm^3 and a temperature of 43 °C. The jet was directed downward for experimental convenience. Food colouring was used in the jet to mimic the behaviour of a non-reactive pollutant. As the dense jet descends, it dilutes with increasing distance from the source due to entrainment of the surrounding water. Upon reaching the floor of the channel, a current is produced which spreads downstream. Measurements of the temperature and salt concentration fields surrounding the jet were conducted using an array of thermocouples and salinity probes, placed regularly along the depth and length of the channel. The concentration of dye was also measured using the image processing and analysis software DigImage [1]. We note that our experiments were conducted with a two-dimensional jet, rather than an axisymmetric source, for convenience.

Figures 1(a), (b) and (c) show the temperature, salinity and dye concentration profiles in the upper layer of water and the pollutant layer spreading on the floor of the channel. We may see that the temperature, salinity and dye concentration are approximately uniform in the vertical direction, in each layer. These results suggest that the eddies generated at the edge of the diffusive interface convectively stir both layers making them well mixed. We expect this behaviour to be valid for axisymmetric jets. We shall use these findings in the development of the theoretical model in the next section.

THEORY

MODELLING OF A BUOYANT JET

We consider a turbulent axisymmetric buoyant jet formed from a discharge of colder and less salty wastewater into the sea of depth H_{total} (Figure 2). We shall assume that the ambient fluid is of infinite lateral extent and motionless. The ambient is not stratified initially and is vertically well mixed by eddies from the diffusive interface after the onset of double diffusion.

We invoke the Boussinesq approximation, i.e. the density variations are small and only important in buoyancy terms. We approximate the density as a linear function of salt concentration and temperature [10]:

$$\rho = 0.99708\,(1 + 0.70181\,S - 2.546\times10^{-4}(T - 25)) \tag{1}$$

Here ρ is density (g/cm^3), S is the mass fraction of salt in solution and T is the temperature (°C).

Equations for conservation of volume, momentum and buoyancy of plumes were first derived by Morton et al.[8]. We extend these conservation statements to a top-hat buoyant jet where density is a function of both temperature and salt concentration:

$$\frac{dQ}{dz} = 2\pi^{1/2}\alpha_e M^{1/2} \tag{2}$$

$$\frac{dM}{dz} = Q\frac{(B_S + B_T)}{M} \tag{3}$$

$$\frac{dB_S}{dz} = 0 \tag{4}$$

$$\frac{dB_T}{dz} = 0 \tag{5}$$

Here Q is the buoyant jet volumetric flowrate; M is the buoyant jet momentum; B_S is the buoyancy flux due to salt; B_T is the buoyancy flux due to temperature and z denotes the vertical co-ordinate, increasing upwards, with its origin at the source. The entrainment constant, α_e, is taken as 0.076 for a strong jet behaviour ($l_M = M_o^{3/4}/B_o^{1/2} > H_{total}$) and 0.117 for a strong plume behaviour (l_M H_{total}); here M_o and B_o are the momentum flux and buoyancy flux of the buoyant jet at the source, respectively.

Since the pollutants are passive in the entrainment process, their concentrations can be calculated by

$$P(z) = \frac{Q(z)\text{-}Q_o}{Q(z)}\times P_1 + \frac{Q_o}{Q(z)}\times P_o \tag{6}$$

where Q_o is the volumetric flowrate from the port and P, P_o and P_1 are the pollutant concentration in the buoyant jet at height z, at the port and in the seawater near the margin of the jet, respectively.

MODELLING THE CLOUD OF POLLUTANT

The wastewater buoyant jet rises to the sea surface and spreads out radially producing a cloud of pollutant. Advection, molecular diffusion and convection due to double diffusion determine the temperature, salt and pollutant concentrations. We shall assume that the depth of the sea (H_{total}) is uniform in the region considered to simplify the problem. The established cloud of pollutant typically has a thickness, H_2, of approximately 15 % of H_{total} [9]. For simplicity, we consider a conservative pollutant (i.e. no decay with time due to predation, sedimentation etc.) here.

Conservation equations

In accord with our experimental observations, we shall assume that the turbulent convection driven by the diffusive interface is strong enough so that the seawater and the cloud of pollutants are well mixed vertically. Hence temperature, salt and pollutant concentrations in both the cloud and the sea are functions of the radial coordinate, r, only. This assumption will give the largest concentration changes and hence will lead to an upper bound for the changes in dilution due to the diffusive interface. This will be useful to assess the impact of the diffusive type interface on wastewater discharges via coastal outfalls.

The depth integrated transport equations for temperature, salt and pollutant are

$$H_2 \frac{\partial c_2}{\partial t} = \left(-\frac{Q_2}{2p} + H_2 \hat{e}_c \right) \frac{1}{r} \frac{\partial c_2}{\partial r} + H_2 k_c \frac{\partial^2 c_2}{\partial r^2} + F_c \qquad (7)$$

$$H_1 \frac{\partial c_1}{\partial t} = \left(\frac{Q_1}{2p} + H_1 \hat{e}_c \right) \frac{1}{r} \frac{\partial c_1}{\partial r} + H_1 k_c \frac{\partial^2 c_1}{\partial r^2} - F_c \qquad (8)$$

where r is the radial coordinate; t is time; H_1 denotes the depth of the sea; Q_1 is the volumetric flowrate of the seawater being entrained into the jet; Q_2 is the volumetric flowrate of the polluted cloud; χ represents T, S and P, i.e. temperature, salt and pollutant concentrations, respectively. κ_χ and F_χ represent the diffusivities and the double diffusive fluxes of the components, T, S and P, respectively. F_χ is taken as positive for a net flux into the surface current. Subscripts 1 and 2 denote the seawater layer and the surface current, respectively.

Flux laws for salt and heat

The fluxes of heat and salt across the diffusive interface have been investigated in many experiments. Turner [12] proposed correlations of the form

$$aF_T = C \left(\frac{g k_T^2}{n} \right)^{1/3} (a \Delta T)^{4/3} \qquad (9)$$

$$bF_S = aF_T R_F \qquad (10)$$

where g is the gravitational constant, ν is the kinematic viscosity, ΔT is the temperature difference between the layers and R_F is called the flux ratio. Kelley[4] constructed best-fit curves for the parameters C and R_F using previous experimental data. We shall adopt his expressions for C and R_F.

$$C = 0.0032 \exp(4.8/R_{\tilde{n}}^{0.72}) \qquad (11)$$

$$R_F = \frac{R_{\tilde{n}} + 1.4(R_{\tilde{n}} - 1)^{3/2}}{1 + 14(R_{\tilde{n}} - 1)^{3/2}} \qquad (12)$$

where R_ρ is stability ratio for the diffusive interface defined as $R_\rho = \beta \Delta S / \alpha \Delta T$.

Pollutant flux law

Unlike for salt and temperature, flux laws for passive components have not been correlated. Therefore, a pollutant flux law will be developed here.

Experimental results [12] show that there are two transport regimes for the heat-salt system at different stability ratios. The ``constant" regime occurs at high stability when the flux ratio is approximately constant. The ``variable" regime describes the rapid change of flux ratio at low stability. Linden[6] argued the increased transport at low stability is due to entrainment across the interface. He modelled the transport of double diffusing components across the diffusive interface as composed of two parts, namely "diffusive flux" and "entrainment flux". The diffusive flux results directly from double diffusive instability. The entrainment flux results from the mechanical mixing across the interface by the convective motions of eddies in the layers. He argued that the two fluxes are independent of one another, hence are additive. We shall extend Linden's [6] model to obtain a flux law for pollutants. The flux of pollutants is composed of two parts, diffusive and entrainment.

$$F_P = u_e \Delta P + F_P^d \qquad (13)$$

Here F_P is the flux of pollutants, u_e is eddy entrainment velocity and $F_P{}^d$ is the ``diffusive" flux of pollutants.

The ratio of flux of pollutants to flux of temperature is:

$$\frac{\text{flux of P}}{\text{flux of T}} = \frac{F_P}{F_T} = \frac{u_e \Delta P + F_P^d}{u_e \Delta T + F_T^d} \tag{14}$$

Adopting Linden's [6] approximations and curve fit results of Kelley [4], the flux of pollutants can be manipulated to be

$$F_P = F_T \times \frac{\Delta P}{\Delta T} \times \frac{1+14\sqrt{k_P / k_T}\,(R_{\tilde{n}}-1)^{3/2}}{1+14(R_{\tilde{n}}-1)^{3/2}} \tag{15}$$

Boundary and initial conditions

Boundary conditions have to be determined to solve the set of 6 PDE's above. At *r=rb*, the buoyant jet spreads out to form the polluted cloud; hence the pollutant cloud has properties equal to those of the buoyant jet. Equation (17) corresponds to the open sea boundary condition at *r=rmax* [5]: an inflow of mass without back return so that the pollutant cloud at large distance from the jet has a uniform diffusive flux in the radial direction.

$$c_2(r = rb, t) = c_{jet}(z = H_{total}, t) \tag{16}$$

$$\left.\frac{\partial}{\partial r}\left(r \frac{\partial c_2}{\partial r} \right)\right|_{r=rmax,t} = 0 \tag{17}$$

For the seawater, the temperature, salt and pollutant concentrations far from the outfall are not affected by the wastewater discharge. Thus the temperature, salt and pollutant concentrations at the shoreline (*r=rmax*) are at their initial background levels and their rate of change with radial position is zero.

$$c_1(r = rmax, t) = c_1(r = rmax, t = 0) \tag{18}$$

$$\left.\frac{\partial c_1}{\partial r}\right|_{r=rmax,t} = 0 \tag{19}$$

The moment the jet reaches the sea surface is taken as t=0. Hence the temperature, salt and pollutant concentrations in the upper layer at t=0 are the same as those for the seawater.

Equations 7 and 8 were solved using Mathematica^{TM}.

RESULTS AND ANALYSIS

In this section, we apply our model to two scenarios of coastal outfalls to assess the importance of double diffusion to pollutant transport. In both scenarios, the outfalls are located 5 km offshore and discharge at a depth of 30 m. The volumetric flowrate is 1 m^3/s and the momentum flux is 4.5 m^4/s^2. Cadmium is chosen as the conservative pollutant. A discharge level of 1 µg/l [14] is chosen and a zero background level is adopted for simplicity. The properties of the buoyant jet and seawater are as in Table 1.

SCENARIO 1: LOW R_p CASE

Some industries discharge effluents with a high salt content to the sea. Examples include desalination plants extracting fresh water from seawater, oil and gas production as a by-product and manufacture or treatment of caustics. In this case, the interface formed between the polluted layer and the seawater has a small ΔS which results in a low interface stability. We model this low interface stability case next.

As the ocean depth (30 m) is less than l_M(38 m), we consider an entrainment constant α_e of 0.076 for a jet. The stability ratio, R_ρ, just after the start of the wastewater discharge is 1.10.

Salt and heat are transported from the seawater below to the overlaying polluted cloud. Thus the salt concentration and temperature in the sea decrease with time (Figures 3 (a) and (b)). Pollutants are transported from the polluted cloud to the seawater leading to an increase in pollutant concentration in the sea with time (Figure 3(c)). When the front of the polluted layer reaches the coast at approximately 240 days, the salt concentration and temperature of the sea close to the jet has decreased by 0.31% and 4.9%, respectively.

In the polluted layer, we expect the temperature and the salt concentration to increase and the pollutant concentration to decrease, comparing with the no double diffusion case, due to the transport by double diffusion. However close to the jet, the opposite is observed (Figures 3 (a), (b) and (c)). This apparent up-gradient transport is due to recirculation. Part of the pollutant excess, temperature deficit and salt deficit are transported from the polluted layer to the seawater due to double diffusion. These are then entrained back into the jet, which carry them to the polluted cloud. The jet entrainment and double diffusion leads to recirculation. At 240 days, the decreases of salt concentration and temperature close to the jet are 0.30% and 4.7%, respectively. The pollutant concentration close to the jet increases by 103% compared to the value in the absence of double diffusion at 240 days. Beyond 1.3 km from the jet, the temperature and salt concentration are higher, and the pollutant concentration is lower, than the corresponding values in the absence of double diffusion.

The stability ratio of the diffusive interface, R_ρ, increases with time close to the buoyant jet, from 1.10 at t=0 to 1.30 at 240 days (Figure 3(d)). R_ρ at the front of the polluted layer current also increases with time. At 240 days, R_ρ is 34, implying the interface at the current front is very stable to double diffusion. Thus, we expect a small transport of salt and pollutant across the interface. This is consistent with the temperature profiles obtained, which show that the temperature difference between the polluted layer and the seawater has almost all diffused away close to the shore. This also explains the almost horizontal region in the salt and pollutant concentration profiles for the polluted layer beyond 1.3 km from the jet.

SCENARIO 2: HIGH R_ρ CASE

The second scenario corresponds to a buoyant jet with a low salt content. Examples include stormwater and some wastewater from sewage treatment works. l_M is found to be 10.3 m and hence we consider an entrainment constant of 0.117, appropriate for a plume.

With a small salt concentration difference between the wastewater and the sea, the resulting diffusive interface is stable and has a high R_ρ. The stability parameter just after the start of the wastewater discharge is 8.3. The changes of the temperature, salinity and pollutant concentration in both layers and the interface stability parameter are qualitatively similar to that in the low R_ρ case, but with much smaller magnitudes (Figures 4 (a), (b), (c) and (d)).

The front of the polluted layer reaches the shore at approximately 135 days. In the sea, the salt concentration and temperature close to the plume decrease by 0.01% and 0.39%, respectively. In the cloud, the pollutant concentration increases by 5.6% close to the plume compared to the value in the absence of double diffusion. The temperature and salt concentration close to the plume decrease by 0.37% and 0.01%, respectively.

DISCUSSION AND CONCLUSIONS

The significance of the recirculation due to double diffusion and the buoyant jet entrainment cannot be captured by mixing zone models adopted by many countries for coastal outfall discharges, since the ``near-field" (the area where the buoyant jet rise and interacts with the ocean surface, bottom and any stratification) and the ``far-field" (the area where ambient environment controls the advection and mixing of the polluted layer) are modelled separately in mixing-zone models.

For a low stability interface between the polluted cloud and the seawater, convection due to double diffusion significantly changes the temperature, salt and especially the pollutant concentrations in both layers, as observed in our first scenario. The pollutant concentration profiles are also very different from those expected in the absence of double diffusion. When the diffusive interface is stable, double diffusion does not significantly alter the salt and pollutant concentrations in either layer.

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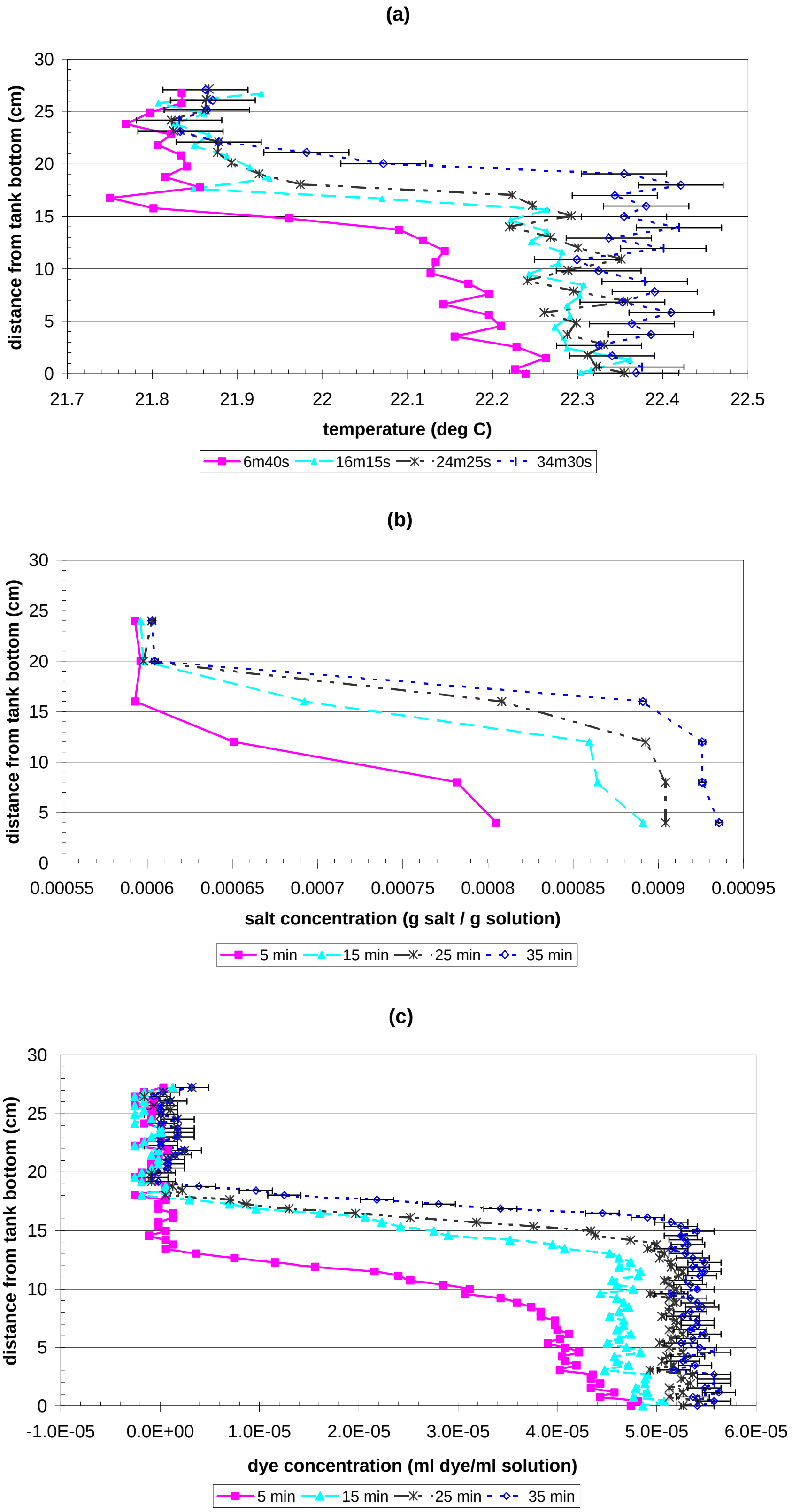


Figure 1: Vertical profiles of (a) salinity, (b) temperature and (c) dye concentration at a position 40 cm downstream of the jet for a low stability interface (R_ρ = 1.6).

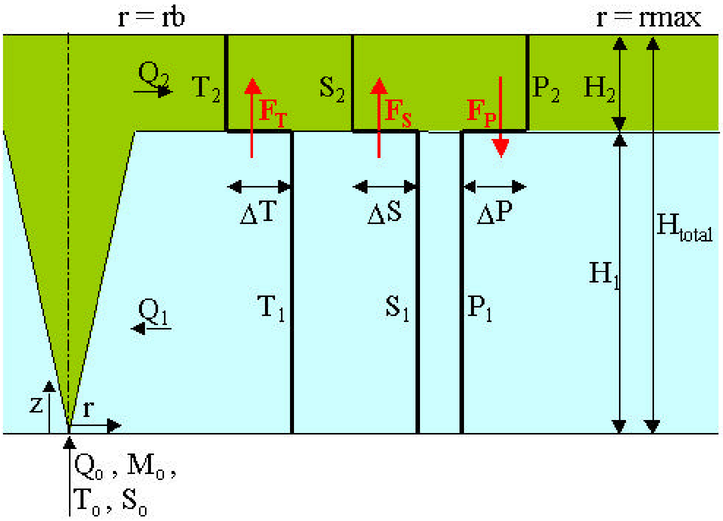
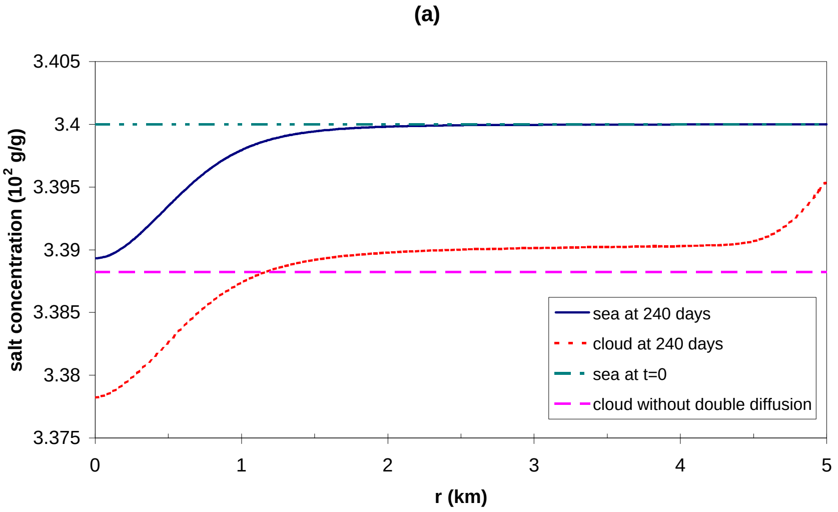


Figure 2: Schematic diagram of the discharge of a wastewater buoyant jet into a calm unstratified sea.


(a)

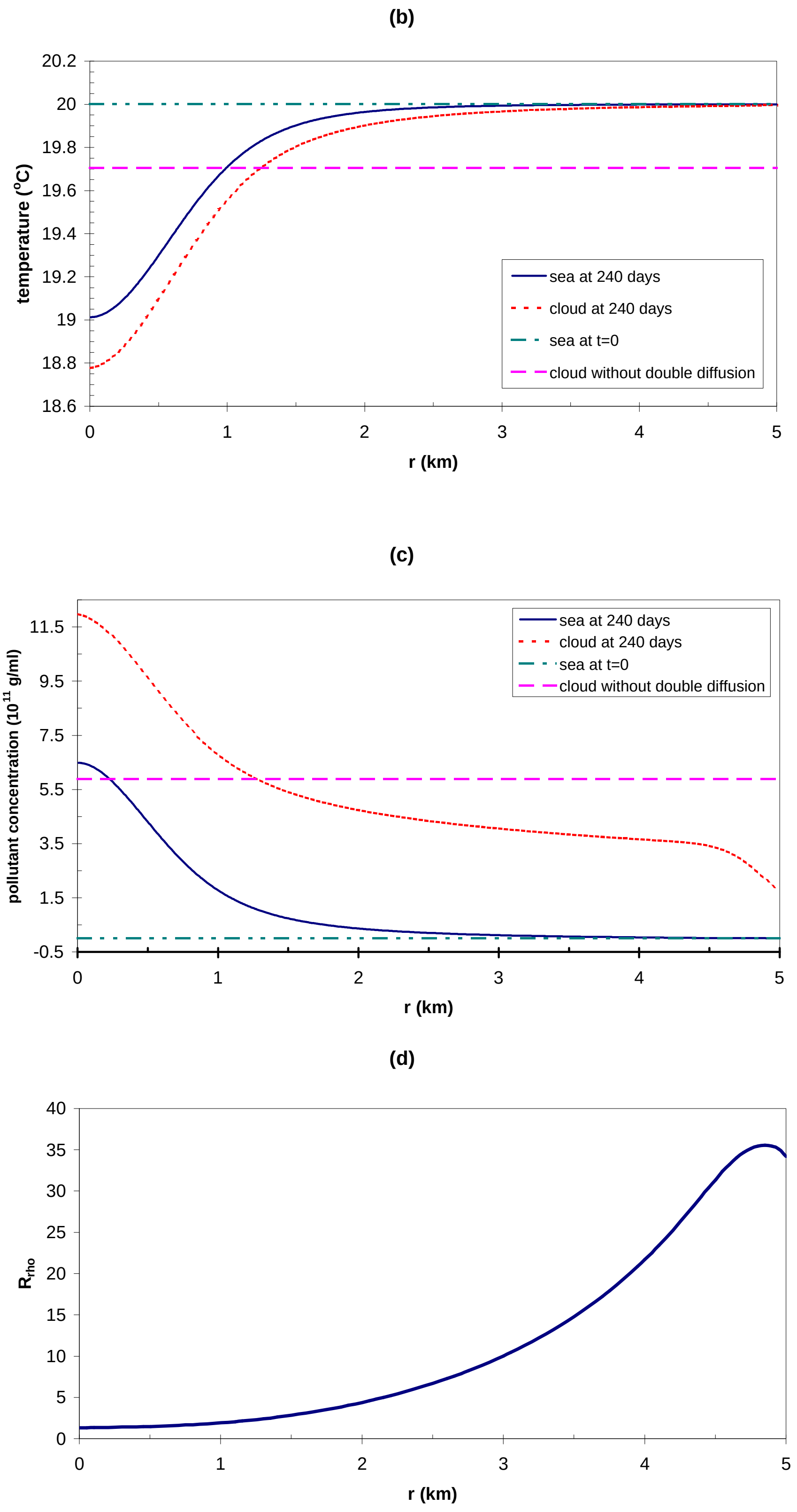
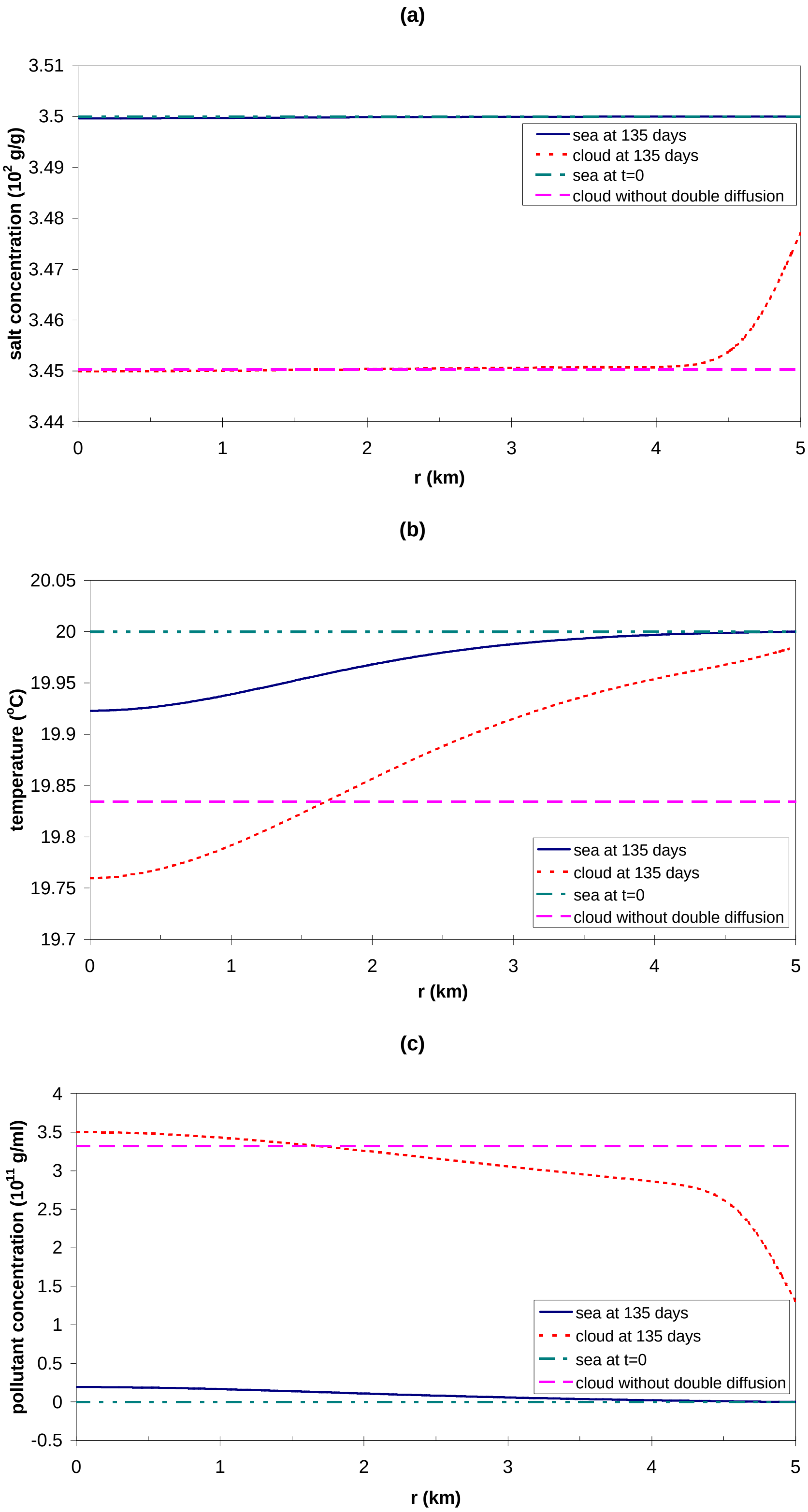


Figure 3: (a) Salinity, (b) temperature, (c) dye concentration and (d) R_ρ profiles for the low stability scenario at 240 days.


(a)
salt concentration (10^2 g/g)
3.51
3.5
3.49
3.48
3.47
3.46
3.45
3.44
0
1
2
3
4
5
r (km)
sea at 135 days
cloud at 135 days
sea at t=0
cloud without double diffusion
(b)
temperature (^{o}C)
20.05
20
19.95
19.9
19.85
19.8
19.75
19.7
0
1
2
3
4
5
r (km)
sea at 135 days
cloud at 135 days
sea at t=0
cloud without double diffusion
(c)
pollutant concentration (10^{11} g/ml)
4
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
0
1
2
3
4
5
r (km)
sea at 135 days
cloud at 135 days
sea at t=0
cloud without double diffusion

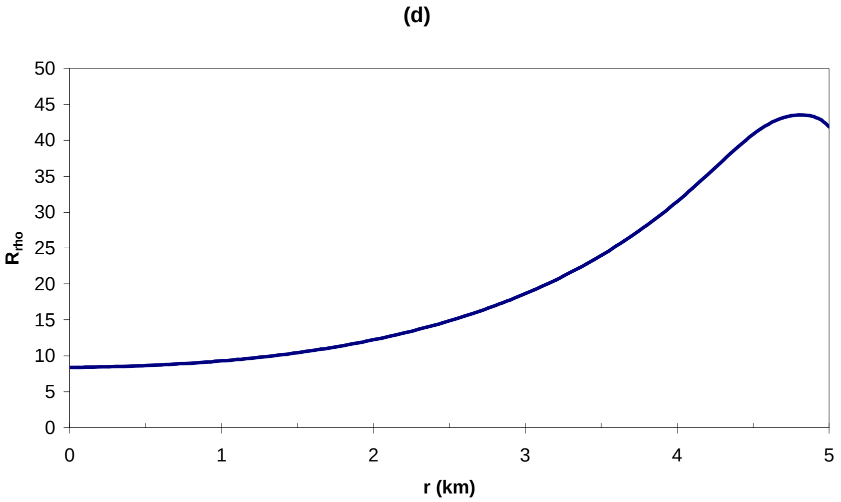


Figure 4: (a) Salinity, (b) temperature, (c) dye concentration and (d) R_ρ profiles for the high stability scenario at 135 days.

		Low R_ρ	High R_ρ
Wastewater buoyant jet	T_o	15 °C	15 °C
	S_o	0.032 g/g	0.020 g/g
	P_o	10^{-9} g/cm^3	10^{-9} g/cm^3
	α_e	0.076	0.117
Seawater	T_1	20 °C	20 °C
	S_1	0.034 g/g	0.035 g/g
	P_1	0 g/cm^3	0 g/cm^3
R_ρ at t=0, r=rb	R_ρ	1.10	8.27

Table 1: Properties of system investigated.