DESIGN PROCEDURE FOR STEEL FRAME STRUCTURES ACCORDING TO BS 5950

2.1 Introduction

Structural design is grossly abbreviated name of an operation, which for major projects may involve the knowledge of hundreds of experts from a variety of disciplines. A code of practice may therefore be regarded as a consensus of what is considered acceptable at the time it was written, containing a balance between accepted practice and recent research presented in such a way that the information should be of immediate use to the design engineer. As such, it is regarded more as an aid to design, which includes allowable stress levels, member capacities, design formulae and recommendations for good practice, rather than a manual or textbook on design.

Once the decision has been taken to construct a particular building, a suitable structural system must be selected. Attention is then given to the way in which loads are to be resisted. After that, critical loading patterns must be determined to suit the purpose of the building. There is, therefore, a fundamental two-stage process in the design operation. Firstly, the forces acting on the structural members and joints are determined by conducting a structural system analysis, and, secondly, the sizes of various structural
members and details of the structural joints are selected by checking against specification member-capacity formulae.

Section 2.2 starts with definitions of basic limit-states terminology. Then, determination of loads, load factors, and load combinations are given as requested by the British codes of practice BS 6399. Accordingly, ultimate limit state design documents of structural elements are described. In this, the discussion is extended to cover the strength, stability and serviceability requirements of the British code of practice BS 5950: Part 1. Finally, the chapter ends by describing methods used, in the present study, to represent the charts of the effective length factor of column in sway or non-sway frames in a computer code.

2.2 Limit state concept and partial safety factors

Limit state theory includes principles from the elastic and plastic theories and incorporates other relevant factors to give as realistic a basis for design as possible. The following concepts, listed by many authors, among them McCormac (1995), Nethercot (1995), Ambrose (1997), and MacGinley (1997), are central to the limit state theory:

1. account is taken of all separate design conditions that could cause failure or make the structure unfit for its intended use,

2. the design is based on the actual behaviour of materials in structures and the performance of real structures, established by tests and long-term observations,

3. the overall intention is that design is to be based on statistical methods and probability theory and

4. separate partial factors of safety for loads and for materials are specified.
The limiting conditions given in Table 2.1 are normally grouped under two headings: ultimate or safety limit states and serviceability limit states. The attainment of ultimate limit states (ULS) may be regarded as an inability to sustain any increase in load. Serviceability limit states (SLS) checks against the need for remedial action or some other loss of utility. Thus ULS are conditions to be avoided whilst SLS could be considered as merely undesirable. Since a limit state approach to design involves the use of a number of specialist terms, simple definitions of the more important of these are provided by Nethercot (1995).

Table 2.1. Limit states (taken from BS 5950: Part 1)

<table>
<thead>
<tr>
<th>Ultimate States</th>
<th>Serviceability states</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Strength (including general yielding, rupture, buckling and transformation into mechanism)</td>
<td>5. Deflection</td>
</tr>
<tr>
<td>2. Stability against overturning and sway</td>
<td>6. Vibration (e.g. wind induced oscillation)</td>
</tr>
<tr>
<td>3. Fracture due to fatigue</td>
<td>7. Repairable damage due to fatigue</td>
</tr>
</tbody>
</table>

Figure 2.1 shows how limit-state design employs separate factors $\gamma_l$, which reflect the combination of variability of loading $\gamma_l$, material strength $\gamma_m$ and structural performance $\gamma_p$. In the elastic design approach, the design stress is achieved by scaling down the strength of material or member using a factor of safety $\gamma_c$ as indicated in Figure 2.1, while the plastic design compares actual structural member stresses with the effects of factored-up loading by using a load factor of $\gamma_p$. 
BS 5950 deliberately adopts a very simple interpretation of the partial safety factor concept in using only three separate $\gamma$ factors:

- Variability of loading $\gamma_l$: loads may be greater than expected; also loads used to counteract the overturning of a structure (see section 2.6) may be less than intended.
- Variability of material strength $\gamma_m$: the strength of the material in the actual structure may vary from the strength used in calculations.
- Variability of structural performance $\gamma_p$: The structure may not be as strong as assumed in the design because of the variation in the dimensions of members, variability of workmanship and differences between the simplified idealisation necessary for analysis and the actual behaviour of the real structure.

A value of 1.2 has been adopted for $\gamma_p$ which, when multiplied by the values selected for $\gamma_l$ (approximately 1.17 for dead load and 1.25 for live load) leads to the values of
\( \gamma_f \), which is then applied to the working loads, examples of which are discussed in Section 2.3. Values of \( \gamma_m \) have been incorporated directly into the design strengths given in BS 5950. Consequently, the designer needs only to consider \( \gamma_f \) values in the analysis.

### 2.3 Loads and load combinations

Assessment of the design loads for a structure consists of identifying the forces due to both natural and designer-made effects, which the structure must withstand, and then assigning suitable values to them. Frequently, several different forms of loading must be considered, acting either singly or in combination, although in some cases the most unfavourable situation can be easily identifiable. For buildings, the usual forms of loading include dead load, live load and wind load. Loads due to temperature effects and earthquakes, in certain parts of the world, should be considered. Other types of structure will have their own special forms of loading, for instance, moving live loads and their corresponding horizontal loads (e.g. braking forces on bridges or braking force due to the horizontal movement of cranes). When assessing the loads acting on a structure, it is usually necessary to make reference to BS 6399: Part 1, 2, and 3, in which basic data on dead, live and wind loads are given.

#### 2.3.1 Dead load

Determination of dead load requires estimation of the weight of the structural members together with its associated “non-structural” component. Therefore, in addition to the bare steelwork, the weights of floor slabs, partition walls, ceiling, plaster finishes and others (e.g. heating systems, air conditions, ducts, etc) must all be considered. Since certain values of these dead loads will not be known until after at least a tentative design
is available, designers normally approximate values based on experience for their initial calculations. A preliminarily value of 6-10 kN/m² for the dead load can be taken.

2.3.2 Live load

Live load in buildings covers items such as occupancy by people, office floor loading, and movable equipment within the building. Clearly, values will be appropriate for different forms of building-domestic, offices, garage, etc. The effects of snow, ice are normally included in this category. BS 6399: Part 1 gives a minimum value of the live load according to the purpose of the structural usage (see BS 6399: Part 1).

2.3.3 Wind load

The load produced on a structure by the action of the wind is a dynamic effect. In practice, it is normal for most types of structures to treat this as an equivalent static load. Therefore, starting from the basic speed for the geographical location under consideration, suitably corrected to allow for the effects of factors such as topography, ground roughness and length of exposure to the wind, a dynamic pressure is determined. This is then converted into a force on the surface of the structure using pressure or force coefficients, which depend on the building’s shape.

In BS 6399: Part 2, for all structures less than 100 m in height and where the wind loading can be represented by equivalent static loads, the wind loading can be obtained by using one or a combination of the following methods:

- standard method uses a simplified procedure to obtain a standard effective wind speed, which is used with standard pressure coefficients to determine the wind loads,
- directional method in which effective wind speeds and pressure coefficients are determined to derive the wind loads for each wind direction.
The outline of the procedure, for calculating wind loads, is illustrated by the flow chart given in Figure 2.2. This shows the stages of the standard method (most widely used method) as indicated by the heavily outlined boxes connected by thick lines. The stages of the directional method are shown as boxes outlined with double lines and are directly equivalent to the stages of the standard method.

The wind loads should be calculated for each of the loaded areas under consideration, depending on the dimensions of the building. These may be:

- the structure as a whole,
- parts of the structure, such as walls and roofs, or
- individual structural components, including cladding units and their fixings.

The standard method requires assessment for orthogonal load cases for wind directions normal to the faces of the building. When the building is doubly-symmetric, e.g. rectangular-plan with flat, equal-duopitch or hipped roof, the two orthogonal cases shown in Figure 2.3 are sufficient. When the building is singly-symmetric, three orthogonal cases are required. When the building is asymmetric, four orthogonal cases are required.

In order to calculate the wind loads, the designer should start first with evaluating the dynamic pressure

\[ q_s = 0.613 V_e^2 \]  

(2.1)

where \( V_e \) is the effective wind speed (m/s), which can be calculated from

\[ V_e = V_s S_b \]  

(2.2)

where \( V_s \) is the site wind speed and \( S_b \) is the terrain and building factor.
Figure 2.2. Flow chart illustrating outline procedure for the determination of the wind loads (according to BS 6399: Part 2)
The terrain and building factor $S_b$ is determined directly from Table 4 of BS 6399: Part 2 depending on the effective height of the building and the closest distance to the sea.

The site wind speed $V_s$ is calculated by

$$V_s = V_b S_a S_d S_s S_p$$

(2.3)

where $V_b$ is the basic wind speed, $S_a$ is the altitude factor, $S_d$ is the directional factor, $S_s$ is the seasonal factor and $S_p$ is the probability factor.

The basic wind speed $V_b$ can be estimated from the geographical variation given in Figure 6 of BS 6399: Part 2.

The altitude factor $S_a$ depends on whether topography is considered to be significant or not, as indicated in Figure 7 of BS 6399: Part 2. When topography is not considered significant, by considering $\Delta_s$, the site altitude (in meters above mean sea level), $S_a$ should be calculated from

$$S_a = 1 + 0.001 \Delta_s.$$  

(2.4)

The directional factor $S_d$ is utilised to adjust the basic wind speed to produce wind speeds with the same risk of being exceeded in any wind direction. Table 3 of BS 6399: Part 2 gives the appropriate values of $S_d$. Generally, when the orientation of the building is unknown, the value of $S_d$ may be taken as 1.

The seasonal factor $S_s$ is employed to reduce the basic wind speed for buildings, which are expected to be exposed to the wind for specific subannual periods. Basically, for permanent buildings and buildings exposed to the wind for a continuous period of more than 6 months a value of 1 should be used for $S_s$. 


The probability factor $S_p$ is used to change the risk of the basic wind speed being exceeded from the standard value annually, or in the stated subannual period if $S_s$ is also used. For normal design applications, $S_p$ may take a value of 1.

The second step of calculating the wind loads is evaluating the net surface pressure $p$ is

$$p = p_e - p_i$$  \hspace{1cm} (2.5)$$

where $p_e$ and $p_i$ are the external and internal pressures acting on the surfaces of a building respectively.

The external pressure $p_e$ is

$$p_e = q_s C_{pe} C_a,$$  \hspace{1cm} (2.6)$$

while the internal surface-pressure $p_i$ is evaluated by

$$p_i = q_s C_{pi} C_a$$  \hspace{1cm} (2.7)$$

where $C_a$ is the size effect factor, $C_{pe}$ is the external pressure coefficient and $C_{pi}$ is the internal pressure coefficient.

Values of pressure coefficient $C_{pe}$ and $C_{pi}$ for walls (windward and leeward faces) are given in Table 5 of BS 6399: Part 2 according to the proportional dimensions of the building as shown in Figure 2.3 and Figure 2.4. In these figures, the building surfaces are divided into different zones (A, B, C, and D). Each of these zone has a different value of $C_{pe}$ and $C_{pi}$ as given in Table 5 of BS 6399: Part 2.

Values of size effect factor $C_a$ are given in Figure 4 of BS 6399: Part 2 in which the diagonal dimension ($a$) of the largest diagonal area to the building category is
considered. The category of the building depends on the effective height of the building and the closest distance between the building and the sea. $C_a$ takes values between 0.5 and 1.0.

The third step of calculating the wind loads is to determine the value of the load $P$ using

$$P = pA.$$  
(2.8)

Alternatively, the load $P$ can be to compute by

$$P = 0.85 (\sum P_i - \sum P_r) (1 + C_r)$$
(2.9)

where $\sum P_i$ is the summation horizontal component of the surface loads acting on the windward-facing walls and roofs, $\sum P_r$ is the summation of horizontal component of the surface loads acting on the leeward-facing walls and roofs and $C_r$ is the dynamic augmentation factor which determined depending on the reference height of the building $H_r$. The factor 0.85 accounts for the non-simultaneous action between faces.
(a) Load cases: wind on long face and wind on short face

(b) Key to pressure coefficient zones

Figure 2.3. Key to wall and flat roof pressure data
Design Procedure for Steel Frame Structures according to BS 5950

Figure 2.4. Key to wall pressure for irregular flush faces (taken from BS 5950)
2.3.4 Load combinations

All relevant loads should be considered separately and in such realistic combinations as to comprise the most critical effects on the elements and the structure as a whole. These combinations are illustrated in BS 5950: Part 1 by the following table.

Table 2.2. Load factors and combinations (taken from BS 5950)

<table>
<thead>
<tr>
<th>Loading</th>
<th>Factor $\gamma_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dead load</td>
<td>1.4</td>
</tr>
<tr>
<td>Dead load restraining uplift or overturning</td>
<td>1.0</td>
</tr>
<tr>
<td>Dead load acting with wind and imposed loads combined</td>
<td>1.2</td>
</tr>
<tr>
<td>Imposed loads</td>
<td>1.6</td>
</tr>
<tr>
<td>Imposed load acting with wind load</td>
<td>1.2</td>
</tr>
<tr>
<td>Wind load</td>
<td>1.4</td>
</tr>
<tr>
<td>Wind load acting with imposed or crane load</td>
<td>1.2</td>
</tr>
<tr>
<td>Forces due to temperature effects</td>
<td>1.2</td>
</tr>
<tr>
<td><strong>Crane loading effects</strong></td>
<td></td>
</tr>
<tr>
<td>Vertical load</td>
<td>1.6</td>
</tr>
<tr>
<td>Vertical load acting with horizontal loads (crabbing or surge)</td>
<td>1.4</td>
</tr>
<tr>
<td>Horizontal load</td>
<td>1.6</td>
</tr>
<tr>
<td>Horizontal load acting with vertical</td>
<td>1.4</td>
</tr>
<tr>
<td>Crane acting with wind load</td>
<td>1.2</td>
</tr>
</tbody>
</table>

The load pattern that should be used to obtain the critical design conditions is not obvious in all cases. The following comments given by MacGinley (1997) are commonly used in the design practice:

- All spans fully loaded should give near the critical beam moments. Other cases could give higher moments.
• The external columns are always bent in double curvature. Full loads will give near critical moments though again asymmetrical loads could give higher values.

• The patterns to give critical moments in the centre column must be found by trials.

2.4 Serviceability limit states

2.4.1 Deflection limits

The deflection under serviceability loads of a building or part should not impair the strength or efficiency of the structure or its components or cause damage to the finishing.

When checking for horizontal and vertical deflections, the most adverse realistic combination and arrangement of serviceability loads should be used, and the structure may be assumed to be elastic. Therefore, BS 5950: Part 1 gives recommended limitations (Δ\text{all} for columns and δ\text{all} for beams) in Table 2.3. These limitations can be expressed by

\[
\frac{|Δ^U_{n_{\text{mem}}} - Δ^L_{n_{\text{mem}}}|}{Δ_{\text{all}}} \leq 1 \quad \text{and} \quad (2.10)
\]

\[
\frac{δ_{\text{max}}}{δ_{\text{all}}} \leq 1 \quad \text{and} \quad (2.11)
\]

where Δ^U_{n_{\text{mem}}} , Δ^L_{n_{\text{mem}}} are the upper and lower horizontal displacements of a column under consideration respectively. The vertical displacement δ_{\text{max}} is the maximum value of within the beam under consideration.

Two alternative loading patterns are requested by BS 5950: Part 1. These are:

• the serviceability loads are taken as the unfactored imposed loads, and
• when considering dead load plus imposed load plus wind load, only 80% of the imposed load and wind need to be considered.

Table 2.3. Deflection limits other than for pitched roof portal frames (see BS 5950)

<table>
<thead>
<tr>
<th>Deflection on beams due to unfactored imposed load</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Cantilevers</td>
<td>Length/180</td>
</tr>
<tr>
<td>Beams carrying plaster or other brittle finish</td>
<td>Span/360</td>
</tr>
<tr>
<td>All other beams</td>
<td>Span/200</td>
</tr>
<tr>
<td>Purlins and sheeting rails</td>
<td>See clause 4.12.2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Deflection on columns other than portal frames due to unfactored imposed load and wind loads</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Tops of columns in single-storey buildings</td>
<td>Height/300</td>
</tr>
<tr>
<td>In each storey of a building with more than one storey</td>
<td>Height of storey under consideration/300</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Crane gantry girders</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical deflection due to static wheel loads</td>
<td>Span/600</td>
</tr>
<tr>
<td>Horizontal deflection (calculated on the top flange properties alone) due to crane surge</td>
<td>Span/500</td>
</tr>
</tbody>
</table>

Note 1. On low-pitched and flat roofs the possibility of ponding needs consideration.
Note 2. For limiting deflections in runway beams refer to BS 2853.

2.4.2 Durability limits

In order to ensure the durability of the structure under conditions relevant to both its intended use and intended life, reference should be made to BS 5493 and BS 4360. Generally, the factors (e.g. the environment, the shape of the members and the structural detailing, whether maintenance is possible, etc) should be considered at the design stage.
2.5 Strength requirements

Situations will often arise in which the loading on a member can not reasonably be represented as a single dominant effect. Such problems require an understanding of the way in which the various structural actions interact with one another. In the simplest cases this may amount to a direct summation of load effects. Alternatively, for more complex problems, careful consideration of the complicated interplay between the individual load components and the resulting deformations is necessary. The design approach discussed in this context is intended for use in situations where a single member is to be designed for a known set of end moments and forces.

Before presenting the strength requirements, a basic idea on classification of sections according to BS 5950 is indicated. This can be presented as follows:

(a) Plastic cross section. A cross section which can develop a plastic hinge with sufficient rotation capacity to allow redistribution of bending moments within the structure.

(b) Compact cross section. A cross section which can develop the plastic moment capacity of the section but in which local buckling prevents rotation at constant moment.

(c) Semi-compact cross section. A cross section in which the stress in the extreme fibres should be limited to yield because local buckling would prevent development of the plastic moment capacity in the section.

(d) Slender cross section. A cross section in which yield of the extreme fibres can not be attained because of premature local buckling.
In order to differentiate between those four types of cross sections, Table 7 of BS 5950: Part 1 illustrates the limiting width to thickness ratios, depending on the type of cross section (e.g. I-section, H-section, circular hollow sections, etc), within which the section can be classified. These limiting ratios are governed by the constant

$$\varepsilon = \sqrt[2]{\frac{275}{p_y}}$$

(2.12)

where \(p_y\) is the design strength, and may be taken as

$$p_y = \begin{cases} 
Y_s & \text{if } Y_s \leq 0.84 U_s \\
0.84 U_s & \text{if } Y_s > 0.84 U_s 
\end{cases}$$

(2.13)

where \(Y_s\) and \(U_s\) are yield strength and ultimate tensile strength. Alternatively, \(p_y\) can be determined according to the flange thickness as given in Table 2.4. In case of slender cross sections, \(p_y\) should be evaluated by multiplying the values given in Table 2.4 by a reduction factor taken from Table 8 of BS 5950: Part 1.

<table>
<thead>
<tr>
<th>Steel Grade</th>
<th>Thickness less than or equal to (mm)</th>
<th>Design strength (p_y) (N/mm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>43</td>
<td>16</td>
<td>275</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>265</td>
</tr>
<tr>
<td></td>
<td>63</td>
<td>255</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>245</td>
</tr>
</tbody>
</table>

**Table 2.4. Design strength**

### 2.5.1 Shear strength

BS 5950: Part 1 requires that the shear force \(F_v\) is not greater than shear capacity \(P_v\). This can be formulated as
\[ \frac{F_x}{P_v} \leq 1. \]  \hspace{1cm} (2.14)

The shear capacity \( P_v \) can be evaluated by

\[ P_v = 0.6 \times p_y \times A_v \]  \hspace{1cm} (2.15)

where \( A_v \) is the shear area and can be taken as given in Table 2.5 considering the cross-sectional dimensions \((t, d, D, B)\) shown in Figure 2.5.

**Table 2.5. Shear area**

<table>
<thead>
<tr>
<th>Section Type</th>
<th>( A_v )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rolled I, H and channel sections</td>
<td>( tD )</td>
</tr>
<tr>
<td>Built-up sections</td>
<td>( td )</td>
</tr>
<tr>
<td>Solid bars and plates</td>
<td>( 0.9A_g )</td>
</tr>
<tr>
<td>Rectangular hollow sections</td>
<td>( [D/(D + B)]A_g )</td>
</tr>
<tr>
<td>Circular hollow sections</td>
<td>( 0.6A_g )</td>
</tr>
<tr>
<td>Any other section</td>
<td>( 0.9A_g )</td>
</tr>
</tbody>
</table>

![Figure 2.5. Dimensions of sections](image)

(a) Circular hollow section (CHS)  
(b) Rolled beams and columns

**Figure 2.5. Dimensions of sections**
When the depth to the thickness ratio of a web of rolled section becomes

\[
\frac{d}{t} \geq 63 \varepsilon ,
\]

(2.16)

the section should be checked for shear buckling using

\[
\frac{F_v}{V_{cr}} \leq 1
\]

(2.17)

where \( V_{cr} \) is the shear resistance that can be evaluated by

\[
V_{cr} = q_{cr} d t .
\]

(2.18)

In this equation, the critical shear strength \( q_{cr} \) can be appropriately obtained from Tables 21(a) to (d) of BS 5950: Part 1.

### 2.5.2 Tension members with moments

Any member subjected to bending moment and normal tension force should be checked for lateral torsional buckling and capacity to withstand the combined effects of axial load and moment at the points of greatest bending and axial loads. Figure 2.6 illustrates the type of three-dimensional interaction surface that controls the ultimate strength of steel members under combined biaxial bending and axial force. Each axis represents a single load component of normal force \( F \), bending about the X and Y axes of the section ( \( M_X \) or \( M_Y \) ) and each plane corresponds to the interaction of two components.

![Figure 2.6. Interaction surface of the ultimate strength under combined loading](image)
2.5.2.1 Local capacity check

Usually, the points of greatest bending and axial loads are either at the middle or ends of members under consideration. Hence, the member can be checked as follows:

\[
\frac{F}{A_e p_y} + \frac{M_X}{M_{CX}} + \frac{M_Y}{M_{CY}} \leq 1
\]  

(2.19)

where \(F\) is the applied axial load in member under consideration, \(M_X\) and \(M_Y\) are the applied moment about the major and minor axes at critical region, \(M_{CX}\) and \(M_{CY}\) are the moment capacity about the major and minor axes in the absence of axial load and \(A_e\) is the effective area.

The effective area \(A_e\) can be obtained by

\[
A_e = \begin{cases} 
K_e A_n & \text{if } K_e A_n \leq A_g \\
A_g & \text{if } K_e A_n > A_g 
\end{cases}
\]  

(2.20)

where \(K_e\) is factor and can be taken as 1.2 for grade 40 or 43, \(A_g\) is the gross area taken from relevant standard tables, and \(A_n\) is the net area and can be determined by

\[
A_n = A_g - \sum_{i=1}^{n} A_i
\]  

(2.21)

where \(A_i\) is the area of the hole number \(i\), and \(n\) is the number of holes.

The moment capacity \(M_{CX}\) can be evaluated according to whether the value of shear load is low or high. The moment capacity \(M_{CX}\) with low shear load can be identified when

\[
\frac{F_v}{0.6P_v} \leq 1.
\]  

(2.22)
Hence, $M_{CX}$ should be taken according to the classification of the cross section as follows:

- for plastic or compact sections

\[
M_{CX} = \begin{cases} \gamma S_X & \text{if } \gamma S_X \leq 1.2 \gamma Z_X \\ 1.2 \gamma Z_X & \text{if } \gamma S_X > 1.2 \gamma Z_X \end{cases}
\]  

(2.23)

- for semi-compact or slender sections

\[
M_{CX} = \gamma Z_X
\]

(2.24)

where $S_X$ is the plastic modulus of the section about the X-axis, and $Z_X$ is the elastic modulus of the section about the X-axis.

The moment capacity $M_{CX}$ with high shear load can be defined when

\[
\frac{F_v}{0.6P_v} > 1.
\]  

(2.25)

Accordingly, $M_{CX}$ is

- for plastic or compact sections

\[
M_{CX} = \begin{cases} \gamma (S_X - S_v \xi) & \text{if } \gamma (S_X - S_v \xi) \leq 1.2 \gamma Z_X \\ 1.2 \gamma Z_X & \text{if } \gamma (S_X - S_v \xi) > 1.2 \gamma Z_X \end{cases}
\]  

(2.26)

where $S_v$ is taken as the plastic modulus of the shear area $A_v$ for sections with equal flanges and

\[
\xi = \frac{2.5 F_v}{P_v} - 1.5.
\]  

(2.27)

- for semi-compact or slender sections

\[
M_{CX} = \gamma Z_X.
\]  

(2.28)

The moment capacity $M_{CY}$ can be similarly evaluated as $M_{CX}$ by using the relevant plastic modulus and elastic modulus of the section under consideration.
2.5.2.2 Lateral torsional buckling check

For equal flanged rolled sections, in each length between lateral restraints, the equivalent uniform moment \( \overline{M} \) is should not exceed the buckling resistance moment \( M_b \). This can be expressed as

\[
\frac{\overline{M}}{M_b} \leq 1.
\] (2.29)

The buckling resistance moment \( M_b \) can be evaluated for members with at least one axis of symmetry using

\[
M_b = p_b S_x
\] (2.30)

where \( p_b \) is the bending strength and can be determined from Table 11 of BS 5950 in which the equivalent slenderness \( \lambda_{LT} \) and \( p_y \) are used.

The equivalent slenderness \( \lambda_{LT} \) is

\[
\lambda_{LT} = n u v \lambda
\] (2.31)

where \( n \) is the slenderness correction factor, \( u \) is the buckling parameters, \( v \) is the slenderness factor and \( \lambda \) is the minor axis slenderness.

The slenderness factor \( v \) can be determined from Table 14 of BS 5950 according to the value of \( \lambda \) and torsional index of the section.

The minor axis slenderness

\[
\lambda = \frac{L_{\text{eff}}}{r_y}
\] (2.32)

should not exceed those values given in Table 2.6, where \( L_{\text{eff}} \) is the effective length, and \( r_y \) is the radius of gyration about the minor axis of the member and may be taken from the published catalogues.
Table 2.6. Slenderness limits

<table>
<thead>
<tr>
<th>Description</th>
<th>Limits</th>
</tr>
</thead>
<tbody>
<tr>
<td>For members resisting loads other than wind loads</td>
<td>$\frac{\lambda}{180} \leq 1$</td>
</tr>
<tr>
<td>For members resisting self weight and wind load only</td>
<td>$\frac{\lambda}{250} \leq 1$</td>
</tr>
<tr>
<td>For any member normally acting as a tie but subject to reversal of stress resulting from the action of wind</td>
<td>$\frac{\lambda}{350} \leq 1$</td>
</tr>
</tbody>
</table>

The equivalent uniform moment $\bar{M}$ can be calculated as

$$\bar{M} = m \times M_A$$  \hspace{1cm} (2.33)

where $M_A$ is the maximum moment on the member or the portion of the member under consideration and $m$ is an equivalent uniform factor determined either by Table 18 of BS 5950: Part 1 or using

$$m = \begin{cases} 
0.57 + 0.33 \beta + 0.10 \beta^2 & \text{if } 0.57 + 0.33 \beta + 0.10 \beta^2 \geq 0.43 \\
0.43 & \text{if } 0.57 + 0.33 \beta + 0.10 \beta^2 < 0.43 
\end{cases}$$  \hspace{1cm} (2.34)

where $\beta$ is the end moment ratios as shown in Figure 2.7.

![Figure 2.7. Determination of the end moment ratio (\(\beta\))](image-url)
2.5.3 Compression members with moments

Compression members should be checked for local capacity at the points of greatest bending moment and axial load usually at the ends or middle. This capacity may be limited either by yielding or local buckling depending on the section properties. The member should also be checked for overall buckling. These checks can be described using the simplified approach illustrated in the following sections.

2.5.3.1 Local capacity check

The appropriate relationship given below should be satisfied.

$$\frac{F}{A_g p_y} + \frac{M_X}{M_{CX}} + \frac{M_Y}{M_{CY}} \leq 1 \quad (2.35)$$

where the parameters $F$, $M_X$, $M_Y$, $A_g$, $p_y$, $M_{CX}$, and $M_{CY}$ are defined in Section 2.5.2.1.

2.5.3.2 Overall buckling check

The following relationship should be satisfied:

$$\frac{F}{A_g p_C} + \frac{m M_X}{M_b} + \frac{m M_Y}{p_y Z_Y} \leq 1 \quad (2.36)$$

where $p_C$ is the compressive strength and may be obtained from Table 27a, b, c and d of BS 5950.

2.6 Stability limits

Generally, it is assumed that the structure as a whole will remain stable, from the commencement of erection until demolition and that where the erected members are incapable of keeping themselves in equilibrium then sufficient external bracing will be
provided for stability. The designer should consider overall frame stability, which embraces stability against sway and overturning.

2.6.1 Stability against overturning

The factored loads considered separately and in combination, should not cause the structure or any part of the structure including its seating foundations to fail by overturning, a situation, which can occur when designing tall or cantilever structures.

2.6.2 Stability against sway

Structures should also have adequate stiffness against sway. A multi-storey framework may be classed as non-sway whether or not it is braced if its sway is such that secondary moments due to non-verticality of columns can be neglected.

Sway stiffness may be provided by an effective bracing system, increasing the bending stiffness of the frame members or the provision of lift shaft, shear walls, etc.

To ensure stability against sway, in addition to designing for applied horizontal loads, a separate check should be carried out for notional horizontal forces. These notional forces arise from practical imperfections such as lack of verticality and should be taken as the greater of:

- 1% of factored dead load from that level, applied horizontally;
- 0.5% of factored dead and imposed loads from that level, applied horizontally.

The effect of instability has been reflected on the design problem by using either the extended simple design method or the amplified sway design method. The extended simple design method is the most used method in which the effective lengths of columns in the plane of the frame are obtained as described in section 2.6.2.2. Determining the effective lengths of columns which is, in essence, equivalent to
carrying out a stability analysis for one segment of a frame and this must be repeated for all segments. The amplified sway design method is based on work done by Horne (1975). In this method, the bending moments due to horizontal loading should be amplified by the factor

\[
\frac{\lambda_{cr}}{\lambda_{cr} - 1}
\]

where \(\lambda_{cr}\) is the critical load factor and can be approximated by

\[
\lambda_{cr} = \frac{1}{200\phi_{s,\text{max}}}
\]

In this equation, \(\phi_{s,\text{max}}\) is the largest value for any storey of the sway index

\[
\phi_{s} = \frac{\Delta^{\text{U}}_{\text{mem}} - \Delta^{\text{L}}_{\text{mem}}}{h_{i}}
\]

where \(h_{i}\) is the storey height, \(\Delta^{\text{U}}_{\text{mem}}\) and \(\Delta^{\text{L}}_{\text{mem}}\) are the horizontal displacements of the top and bottom of a column as shown in Figure 2.5.

In amplified sway design method, the code requires that the effective length of the columns in the plane of the frame may be retained at the basic value of the actual column length.

### 2.6.2.1 Classification into sway /non-sway frame

BS 5950: Part 1 differentiates between sway and non-sway frames by considering the magnitude of the horizontal deflection \(\Delta_{i}\) of each storey due to the application of a set of notional horizontal loads. The value of \(\Delta_{i}\) can therefore be formulated as

\[
\Delta_{i} = \left| \Delta^{\text{U}}_{\text{mem}} - \Delta^{\text{L}}_{\text{mem}} \right|
\]

This reflects the lateral stiffness of the frame and includes the influence of vertical loading. If the actual frame is unclad or is clad and the stiffness of the cladding is taken
into account in the analysis, the frame may be considered to be non-sway if \( \Delta_i \), for every storey, is governed by

\[
\Delta_i \leq \frac{h_i}{4000}.
\]  

(2.41)

If, as in frequently the case, the framework is clad but the analysis is carried out on the bare framework, then in recognition of the fact that the cladding will substantially reduce deflections, the condition is relaxed somewhat and the frame may be considered as non-sway if the deflection of every storey is

\[
\Delta_i \leq \frac{h_i}{2000}.
\]  

(2.42)

### 2.6.2.2 Determination of the effective length factor

Work by Wood (1974a) has led to the development of few ‘simple-to-use’ charts, which permit the designer to treat the full spectrum of end restraint combinations. The approach is based upon the considerations of a limited frame as indicated in Figure 2.9a where \( K_U \), \( K_L \), \( K_{TL} \), \( K_{TR} \), \( K_{BL} \), and \( K_{BR} \) are the values for \( IIL \) for the adjacent upper, lower, top-left, top-right, bottom-left and bottom right columns respectively.

For non-sway frames, the analysis is based upon a combination of two possible distorted components-joint rotations at the upper and lower end of the column under consideration and the calculation of the elastic critical load using the stability functions. For structures in which horizontal forces are transmitted to the foundations by bending moments in the columns, a similar limited frame is considered as shown in Figure 2.9b.

For non-sway frames, the solution to the stability criterion is plotted as contours directly in terms of effective length ratio as shown in Figure 23 of BS 5950. On the other hand, for sway structures, the analysis from which the chart (Figure 24 of BS
Design Procedure for Steel Frame Structures according to BS 5950

5950) is derived has considered not only rotations at the column ends but also the freedom to sway. Utilising the restraint coefficients $k_1$ and $k_2$ evaluated from

$$k_1 = \frac{K_C + K_U}{K_C + K_U + K_{TL} + K_{TR}} \quad \text{and} \quad k_2 = \frac{K_C + K_L}{K_C + K_L + K_{BL} + K_{BR}}.$$  \hspace{1cm} (2.43)

The value of the effective length factor $L_{eff}^X / L$ may be interpolated from the plotted contour lines given in either Figure 23 of BS 5950 established for a column in non-sway framework or Figure 24 of BS 5950 prepared for a column in sway framework.

(a) Limited frame for a non-sway frame
(b) Limited frame for a sway frame

\textbf{Figure 2.9.} Limited frame for a non-sway and sway frames

When using Figure 23 and 24 of BS 5950, the following points should be noted.

1. Any member not present or not rigidly connected to the column under consideration should be allotted a $K$ value of zero.
2. Any restraining member required to carry more than 90% of its moment capacity (reduced for the presence of axial load if appropriate) should be allotted a $K$ value of zero.

3. If either end of the column being designed is required to carry more than 90% of its moment-carrying capacity the value of $k_1$ or $k_2$ should be taken as 1.

4. If the column is not rigidly connected to the foundation, $k_2$ is taken as minimal restraint value equal to 0.9, unless the column is actually pinned (thereby deliberately preventing restraint), when $k_2$ equals to 1.

5. If the column is rigidly connected to a suitable foundation (i.e. one which can provide restraint), $k_2$ should be taken 0.5, unless the foundation can be shown to provide more restraint enabling a lower (more beneficial) value of $k_2$ to be used.

The first three conditions reflect the lack of rotational continuity due to plasticity encroaching into the cross-section and the consequent loss of elastic stiffness, which might otherwise be mobilized in preventing instability.

Three additional features, which must be noted when employing these charts, namely the value of beam stiffness $K_b$ to be adopted. In the basic analysis from which the charts were derived it was assumed that the ends of the beams remote from the column being designed were encastre (Steel Construction Institute, 1988). This is not however always appropriate and guidance on modified beam stiffness values is listed as follows:

1. For sway and non-sway cases, for beams, which are directly supporting a concrete floor slab, it is recommended that $K_b$ should be taken as $I_b/L$ for the member.
2. For a rectilinear frame not covered by concrete slab and which is reasonably regular in layout:

a) For non-sway frames, where it can be expected that single curvature bending will occur in the beam as indicated Figure 2.10a, $K_b$ should be taken as $0.5I_b/L$.

b) For sway frames, where it can be expected that double-curvature bending will exist as indicated in Figure 2.10b, the operative beam stiffness will be increased and $K_b$ should be taken as $0.5I_b/L$. A rider is added to this clause which notes that where the in-plane effective length has a significant influence on the design then it may be preferable to obtain the effective lengths from the critical load factor $\lambda_{cr}$. This reflects the approximate nature of the charts and the greater accuracy possible from critical load calculation.

3. For structures where some resistance to sidesway is provided by partial sway bracing or by the presence of infill panels, two other charts are given. One for the situation where the relative stiffness of the bracing to that of the structure denoted by $k_3$ is 1 and the second where $k_3 = 2$.

(a) Buckling mode of non-sway frame  (b) Buckling mode of sway frame

*Figure 2.10. Critical buckling modes of frame (taken from BS 5950)*
2.7 Flowchart of design procedure

Figure 2.11 shows the limit state design procedure for a structural framework.

Start

Apply notional horizontal loading condition, compute horizontal nodal displacements and determine whether the framework is sway or non-sway

Compute the effective buckling lengths

Apply loading condition $q = 1, 2, \Lambda, Q$: if the framework is sway, then include the notional horizontal loading condition

Analyze the framework, compute normal force, shearing force and bending moments for each member of the framework

Design of member $n_{\text{mem}} = 1, 2, \Lambda, N_{\text{mem}}$

Determine the type of the section (compact or semi-compact or compact or slender) utilising Table 7 of BS 5950

Evaluate the design strength of the member

Check the slenderness criteria

Tension member? Yes

End

Figure 2.11a. Flowchart of design procedure of structural steelwork
Figure 2.11b. (cont.) Flowchart of design procedure of structural steelwork
2.8 Computer based techniques for the determination of the effective length factor

The values of the effective length factor given in Figures 23 and 24 of BS 5950 are obtained when the condition of vanishing determinantal form of the unknown of the equilibrium equations is satisfied (see Chapter 3). The slope deflection method for the stability analysis is used (see Wood 1974a). This method is based on trial and error technique. Accordingly, it is difficult to link such this method to design optimization algorithm. Therefore, to incorporate the determination of the effective length factor $L_X^{\text{eff}}/L$ for a column in either sway or non-sway framework into a computer based algorithm for steelwork design, two techniques are presented. These techniques are described in the following sections.

2.8.1 Technique 1: Digitizing the charts

It can be seen that each of these figures is symmetric about one of its diagonal thus, digitizing a half of the plot will be sufficient. The following procedure is applied.

Step 1. The charts have been magnified to twice their size as given in BS 5950: Part 1.

Step 2. Each axis ($k_1$ and $k_2$) is divided into 100 equal divisions, resulting in 101 values \{0.0, 0.01, ..., 1.0\} for each restraining coefficient.

Step 3. More contour lines have been added to get a good approximation for each chart.

Step 4. The value of $L_X^{\text{eff}}/L$ was read for each pair ($k_1$, $k_2$) of values according to the division mentioned above. These are graphically represented as shown in Figures 2.12 and 2.13 for non-sway and sway charts respectively. In Figure 2.13, values of $L_X^{\text{eff}}/L$ greater than 5 are not plotted.
Step 5. Two arrays, named $S$ for sway and $NS$ for non-sway, are created each of them is a two-dimensional array of 101 by 101 elements. These arrays contain the 10201 digitized values of $L_{X}^{\text{eff}}/L$.

Step 6. The calculated values ($k_1$ and $k_2$) applying equation (2.43) are multiplied by 100, and the resulting values are approximated to the nearest higher integer numbers named $k_1^h$ and $k_2^h$. The value of $L_{X}^{\text{eff}}/L$ may then be taken from the arrays as

$$\frac{L_{X}^{\text{eff}}}{L} = \begin{cases} 
S(k_1^h + 1, k_2^h + 1) & \text{for a column in a sway frame} \\
NS(k_1^h + 1, k_2^h + 1) & \text{for a column in a non-sway frame}. 
\end{cases}$$

(2.44)
**Figure 2.12.** Surface plot of the restraint coefficients $k_1$ and $k_2$ versus $\frac{L_{X}^{\text{eff}}}{L}$ for a column in a rigid-jointed non-sway frame

**Figure 2.13.** Surface plot of the restraint coefficients $k_1$ and $k_2$ versus $\frac{L_{X}^{\text{eff}}}{L}$ for a column a rigid-jointed sway frame
2.8.2 Technique 2: Analytical descriptions of the charts

In this section, polynomials are obtained applying two methodologies. Firstly, a statistical package for social sciences named SPSS is used. Secondly, the genetic programming methodology is employed.

2.8.2.1 Regression analysis

In this section, polynomials are assumed and their parameters are obtained by applying the Levenberg-Marquart method modeled in SPSS (1996). Several textbooks discuss and give FORTRAN code for this method among them Press et al. (1992). SPSS is a general-purpose statistical package, which is used for analysing data. In this context, pairs of \((k_1, k_2)\) and their corresponding \(\frac{L_{\text{eff}}}{L}\) values are used to form polynomials, which can then be tested. A guide to data analysis using SPSS is given by Norusis (1996). The procedure used can be described as follows:

Step 1. The charts are magnified to twice as their size as established in BS 5950.

Step 2. Each axis \((k_1\) and \(k_2)\) are divided into 20 equal divisions. This results in 21 values \([0.0, 0.05, \ldots, 1.0]\) for each axis. More contour lines are then added to each chart. Hence, the value of \(\frac{L_{\text{eff}}}{L}\) was approximated for each pair \((k_1, k_2)\) according to the division given above.

Step 3. The data SPSS data file is prepared. Here, the dependent \(\frac{L_{\text{eff}}}{L}\) and independent \((k_1, k_2)\) variables and the analysis type (linear or nonlinear) is chosen.

Step 4. A general shape of a polynomial is assumed. The general forms

\[
\frac{L_{\text{eff}}}{L} = a_0 + a_1 k_1 + a_2 k_2 + a_3 k_1^2 + a_4 k_2^2 + a_5 k_1 k_2,
\]  

(2.45)
\[
\frac{L_{\text{eff}}}{L} = a_0 + a_1 k_1^2 + a_2 k_2^2 + a_3 k_1 k_2 + a_4 k_1^2 k_2 + a_5 k_1 k_2 + \ldots
\]
and
\[
\frac{L_{\text{eff}}}{L} = a_0 + a_1 k_1^2 + a_2 k_2^2 + a_3 k_1 k_2 + a_4 k_1^2 k_2 + a_5 k_1 k_2 + \ldots
\]

are utilised to express the 2nd, 3rd and 4th order polynomials, where \(a_0, a_1, \ldots, a_n\) are the parameters.

**Step 5.** Carrying out the analysis by examining relationships between the dependent and independent variables and testing the hypotheses (see Norusis, 1996).

**Step 6.** Accepting the obtained results or repeat steps 4 and 5.

When applying the proposed technique to achieve polynomials of \(L_{\text{eff}}/L\) of a column in sway framework, a poor performance of testing the hypotheses is performed because the infinity values of \(L_{\text{eff}}/L\) is estimated. This occurs when either of the column ends becomes pinned. Thus, values of \(L_{\text{eff}}/L\) greater than 5 are truncated and the technique is repeated.

For a column in non-sway framework, the obtained polynomials of \(L_{\text{eff}}/L\) are listed in Table 2.7. Table 2.8 comprises polynomials for a column in sway framework. The 4th order polynomials are depicted as shown in Figures 2.14 and 2.15 for a column in non-sway and sway framework respectively. The sum of square of the differences \(SUM_{\text{diff}}\) between the calculated \((L_{\text{eff}}/L)_{i,\text{cal}}\) and read \((L_{\text{eff}}/L)_{i,\text{dig}}\) is

\[
SUM_{\text{diff}} = \sum_{i=1}^{44} \left[ \frac{L_{\text{eff}}}{L} \right]_{i,\text{cal}} - \left[ \frac{L_{\text{eff}}}{L} \right]_{i,\text{dig}}^2
\]  
(2.48)
Table 2.7. Polynomials obtained for $L_{X}^{\text{eff}}/L$ of a column in a non-sway frame

<table>
<thead>
<tr>
<th>Equation</th>
<th>$SUM^{\text{diff}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_{X}^{\text{eff}}/L = 0.50562 + 0.11803 k_1 + 0.11803 k_2 +$</td>
<td>0.00025</td>
</tr>
<tr>
<td>$0.0772 k_1^2 + 0.0772 k_2^2 + 0.094815 k_1 k_2$</td>
<td></td>
</tr>
<tr>
<td>$L_{X}^{\text{eff}}/L = 0.499363 + 0.134267 k_1 + 0.134267 k_2 + 0.1098 k_1^2 +$</td>
<td>0.000736</td>
</tr>
<tr>
<td>$0.1098 k_2^2 - 0.0302258 k_1 k_2 + 0.06252 k_1^2 k_2 +$</td>
<td></td>
</tr>
<tr>
<td>$0.06252 k_1 k_2^2 - 0.042576 k_1^3 - 0.042576 k_2^3$</td>
<td></td>
</tr>
<tr>
<td>$L_{X}^{\text{eff}}/L = 0.51496 + 0.1129124 k_1 + 0.1129124 k_2 +$</td>
<td>0.000579</td>
</tr>
<tr>
<td>$0.185809 k_1^2 + 0.185809 k_2^2 - 0.0005816 k_1 k_2 +$</td>
<td></td>
</tr>
<tr>
<td>$0.015352 k_1 k_2 + 0.015352 k_1 k_2^2 - 0.1343541 k_1^3 -$</td>
<td></td>
</tr>
<tr>
<td>$0.1343541 k_2^3 + 0.032573 k_1^4 + 0.032573 k_2^4 +$</td>
<td></td>
</tr>
<tr>
<td>$0.053263 k_1^3 k_2 + 0.053263 k_1 k_2^3 - 0.032727 k_1^2 k_2^2$</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.8. Polynomials obtained for $L_{X}^{\text{eff}}/L$ of a column in a sway frame

<table>
<thead>
<tr>
<th>Equation</th>
<th>$SUM^{\text{diff}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_{X}^{\text{eff}}/L = 0.78173 + 1.78797 k_1 + 1.78797 k_2 - 2.93426 k_1^2 -$</td>
<td>1.8086</td>
</tr>
<tr>
<td>$2.93426 k_2^2 - 3.822909 k_1 k_2 + 2.629751 k_1^3 k_2 +$</td>
<td></td>
</tr>
<tr>
<td>$2.629751 k_1 k_2^2 + 2.36469 k_1^3 + 2.36469 k_2^3$</td>
<td></td>
</tr>
<tr>
<td>$L_{X}^{\text{eff}}/L = 1.14112 - 1.32226 k_1 - 1.32226 k_2 + 5.5268 k_1^2 +$</td>
<td>0.555406</td>
</tr>
<tr>
<td>$5.5268 k_2^2 + 7.00754 k_1 k_2 - 10.41008 k_1^2 k_2 -$</td>
<td></td>
</tr>
<tr>
<td>$10.41008 k_1 k_2^2 - 6.91641 k_1^3 - 6.91641 k_2^3 +$</td>
<td></td>
</tr>
<tr>
<td>$3.579345 k_1^4 + 3.579345 k_1^4 + 5.22176 k_1^3 k_2 +$</td>
<td></td>
</tr>
<tr>
<td>$5.22176 k_1 k_2^3 + 5.89349 k_1^2 k_2^2$</td>
<td></td>
</tr>
</tbody>
</table>
Figure 2.14. Graphical representation of the 4th order polynomial of $L_X^{\text{eff}}/L$ for a column in a rigid-jointed non-sway frame.
Figure 2.15. Graphical representation of 4\textsuperscript{th} order polynomial of $\frac{L_{X}^{\text{eff}}}{L}$ for a column in a rigid-jointed non-sway frame
From Figure 2.14, it can be observed that the 4th order polynomial obtained for the determination of $L_{X}^{\text{eff}}/L$ of a column in non-sway framework is almost identical to the original data that is shown in Figure 2.12. It can be also deduced from Figure 2.15 that that the 4th order polynomial obtained for the determination of $L_{X}^{\text{eff}}/L$ of a column in sway framework is not in a good agreement with the original data shown in Figure 2.13.

### 2.8.2.2 Genetic programming (GP)

This section describes the analytical description of Figures 23 and 24 of BS 5950. Toropov and Alvarez (1998) suggested the use of GP methodology (Koza, 1992) for the selection of the structure of an analytical approximation expression. The expressions are composed of elements from a terminal set (design variables $k_1$ and $k_2$), and a functional set (mathematical operators +, *, /, -, etc), which are known as nodes. The functional set, as shown in Figure 2.16, can be subdivided into binary nodes, which take any two arguments, and unary nodes, which take one argument e.g. square root.

![Typical tree structure](image)

**Figure 2.16.** Typical tree structure representing $a_0 + (a_1 k_1 + a_2 k_2)^2$

Modelling the expression evolves through the action of three basic genetic operations: reproduction, crossover and mutation. In the reproduction stage, $N_p$
expressions are randomly created and the fitness of each expression is calculated. A strategy must be adopted to decide which expressions should die. This is achieved by killing those expressions having a fitness below the average fitness. Then, the population is filled with the surviving expressions according to fitness proportionate selection. New expressions are then created by applying crossover and mutation, which provide diversity of the population. Crossover (Figure 2.17) combines two trees (parents) to produce two children while mutation (Figure 2.18) protects the model against premature convergence and improves the non-local properties of the search.

![Diagram of Crossover](image-url)

**Figure 2.17.** Crossover
The GP methodology has been applied where the 4th order polynomial

\[
\frac{L_{\text{eff}}}{L} = 0.51496 + 0.1129124 k_1 + 0.1129124 k_2 + 0.185809 k_1^2 + 0.185809 k_2^2 - 0.0005816 k_1 k_2 + 0.015352 k_1^2 k_2 + 0.015352 k_1 k_2^2 - 0.1343541 k_1^3 - 0.1343541 k_2^3 + 0.032573 k_1^4 + 0.032573 k_2^4 + 0.053263 k_1^3 k_2 + 0.053263 k_1 k_2^3 - 0.03272728 k_1^2 k_2^2
\]

is obtained for the determination of the effective length factor of a column in non-sway framework. For a column in sway framework, the 7th order polynomial

\[
\frac{L_{\text{eff}}}{L} = 0.982287 + 0.843802 k_1 + 0.843802 k_2 - 4.70571 k_1^2 - 4.70571 k_2^2 - 9.269518 k_1 k_2 + 45.2545 k_1^2 k_2 + 45.2545 k_1 k_2^2 + 19.12882 k_1^3 + 19.12882 k_2^3 - 39.380878 k_1^4 - 39.380878 k_2^4 - 103.84266 k_1^3 k_2 - 103.84266 k_1 k_2^3 - 140.678021 k_1^2 k_2^2 + 47.49748 k_1^5 + 47.49748 k_2^5 + 124.82327 k_1 k_2^4 + 124.82327 k_1^4 k_2 + 202.337 k_1^3 k_2^3 + 202.337 k_1^3 k_2^3 - 31.31407 k_1^6 - 31.31407 k_2^6 - 75.925149 k_1 k_2^5 - 75.925149 k_1^5 k_2 - 141.37368 k_1^4 k_2^2 - 141.37368 k_1^2 k_2^4 - 164.73689 k_1^3 k_2^3 + 8.97372 k_1^7 + 8.97372 k_2^7 + 18.5579 k_1^6 k_2 + 18.5579 k_1 k_2^6 + 39.4783 k_1^3 k_2^5 + 39.4783 k_1^5 k_2^3 + 49.9215 k_1^4 k_2^4 + 49.9215 k_1^2 k_2^6
\]

is achieved. This expression is depicted in Figure 2.19.

Figure 2.18. Mutation
Figure 2.19. Graphical representation of 7th order polynomial of $L^\text{eff}_X/L$ for a column in a rigid-jointed non-sway frame.
2.9 Concluding remarks

This chapter reviewed an overall background to the concept of the limit state design for steel framework members to BS 5950. A brief introduction to the applied loads and load combination is presented. The serviceability, strength as well as sway stability criteria employed in the present context are summarised. Furthermore, a general layout of the developed computer-based technique to framework design is presented. Finally, analytical polynomials are achieved to automate the determination of the effective length factor. In this investigation, the following observations can be summarised.

- For columns in a non-sway framework, the $2^{nd}$ or $3^{rd}$ or $4^{th}$ order polynomials presented in Table 2.8 can be successfully used to determine the effective length factor in design optimization based techniques.

- For columns in a sway framework, the $7^{th}$ order polynomial can be applied when both of the column restraint coefficients are less than 0.95. Otherwise, the technique discussed in Section 2.8.1 may be utilised.

This chapter now will be followed by theory and methods of computing the critical buckling load employed for more accurate evaluation of the effective buckling length.