Structural Mechanics – summary and problems

Introduction

The subject is the study of **stresses** and **strains** in bodies. Knowledge of stresses and strains is needed in the design of engineering components. The subject is also known as Solid Mechanics, Mechanics of Deformable Bodies, Mechanics of Materials or Strength of Materials.

Text book:

Mechanics of Engineering Materials Benham, Crawford and Armstrong 2nd edition Longman 1996.

Outline and preliminary timetable:

Wk	Chapter	Content	Pages in textbook
1 and 2	1	Introduction. Statically determinate *force* systems (revision).	1-14; 16-18; 21-32
3	2	Statically determinate *stress* systems.	43-46; 54-58
4	3	Stress-strain relations.	64-74
5	4	Statically indeterminate stress systems.	82-84; 88-91
6	Appendix A	Properties of areas (revision).	598-601
7 and 8	6	Bending - stress.	125-137; 147-151
9	7	Bending - slope and deflection.	185-186; 193-202
10 and 11	8	Stress and strain transformations.	292-296; 298-301
11 and 12		Revision	

Chapter 1 Statically-determinate force systems

Objectives:

- understand essential concepts of statics, draw free-body diagrams and solve for reaction forces and moments.

- understand concepts of loading, support and sign conventions for bending and draw shear force and bending moment diagrams.

- understand various other loading conditions and the principle of superposition.

1.1 Revision of statics (pp 1-14; 16-18)

- importance of statics
- equilibrium conditions and equations
- force and moment resolution
- types of structural and solid body components and supports
- free body diagram: internal and external forces, determinacy and pin-jointed frames
- directions and signs

1.2 Bending - types of loading, support and sign convention (pp 21-24)

- types of loading
- types of support
- determinacy
- force and moment equilibrium
- sign convention

1.3 Shearing force (SF) and bending moment (BM) diagram (pp 24-28)

- definition
- examples
- summary of procedures
- point of contraflecture

1.4 Other loading conditions and principle of superposition (pp 28-32)

- torsion of members
- combination of axial force, bending and torque
- principle of superposition

Note 1 - Direction and signs

- choose a co-ordinate system
- any direction of force or moment can be assumed for the free body diagram
- write equilibrium equations according to the chosen co-ordinate system
- solve for unknown forces and moments
- if the sign of the solution is positive, then the direction of the force or moment is the same as that assumed
- if it is negative, then the direction of the force or moment is the opposite to that assumed

Note 2 - procedures for drawing SF and BM diagrams

1. Determination of reactions on the beam using static equilibrium

2. Determination of SF and BM at different sections divided by

- point loads
- couples
- supports

3. Calculate SF and BM at critical points

- ends
- point loads
- couples
- zero shear stress
- maximum bending moments

4. How SF and BM vary depends external loading

SF stepwise if no distributed loads
linear for uniformly distributed loads
BM linear if no distributed loads
parabolic for uniformly distributed loads

5. Check points

SF abrupt changes in value at supports or concentrated loads, with the magnitude of change equal to the load acting
BM zero at simply supported ends
changes in value at a couple, with the change equal to the magnitude of the couple

6. Relationships

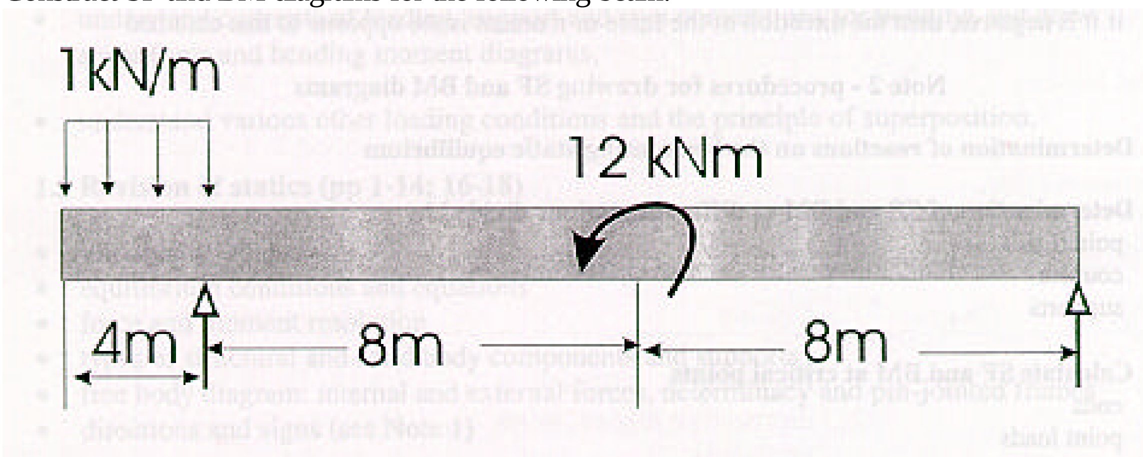
- introduction of BM does not cause any change in SF
- maximum BM at point loads, supports, at a cross-section where SF equals zero or at a couple
- change in SF equals the area under the distributed load
- change in BM equals the area under the SF diagram (not valid at concentrated loads or couples)

Chapter 1 problems

Attempt 1.6, 1.8, and 1.28 from the textbook.

Attempt the following further problem:

Construct SF and BM diagrams for the following beam:



Chapter 2 Statically-determinate stress systems

Objectives:

- understand important concepts of stress - normal stress and shear stress, with associated sign conventions.

- analyse and solve simple stress systems.

2.1 Concepts, definition and sign conventions (pp 43-45)

- stress
- normal stress
- shear stress
- stress element
- shear stress relations
- hydrostatic stress

2.2 Introduction to statically determinate stress systems (pp 45-46)

- assumptions/approximations
- solution procedures

2.3 Bar and strut (column) (p 46)

2.4 Thin sphere (p 54)

2.5 Thin cylinder (pp 54-56)

2.6 Simple shear (p 57)

2.7 Torsion of a thin circular tube (pp 57-58)

Chapter 2 problems

Attempt 2.1, 2.9, 2.14 and 2.19 from the textbook.

Attempt the following further problem:

Show that the axial force P producing uniform tension or compression in a prismatic member must act through the centre of the cross-section.

Chapter 3 Stress-strain relations

Objectives:

- understand important concepts of deformation and strain including normal, shear and volumetric strain
- understand stress-strain relations for various engineering materials
- understand the concept of various material constants
- apply general stress-strain relations to solve simple problems

3.1 Deformation (p 64)

- concept
- design consideration
- various types of deformation

3.2 Strain (pp 64-67)

- normal strain
- shear strain
- volumetric strain

3.3 Material deformation (p 67)

3.4 Stress-strain relations (pp 67-70)

- **Hooke's Law**
- normal stress and strain, elastic modulus
- shear stress and strain, shear modulus
- hydrostatic stress and volumetric strain, bulk modulus
- lateral strain and Poisson's ratio

3.5 Thermal strain (pp 70-71)

3.6 General stress-strain relations (pp 71-74)

- equations
- plane stress and plane strain

3.7 Strains in a statically determinate problem (pp 74-75)

Chapter 3 problems

Attempt 3.1, 3.12 from the textbook.

Attempt the following further problem:

A thin cylinder with closed ends has a wall thickness of 4 mm, an inside diameter of 80 mm and is subject to an internal pressure of 20 MPa. E = 207 GPa and ν = 0.3. Assume zero radial stress and calculate the circumferential stress σ_y, the longitudinal stress σ_x, the corresponding strains and the volumetric strain.
[Answers σ_y = 200 MPa; σ_x = 100 MPa; volumetric strain = 579.7 $\times$ 10-6.]

Chapter 4 Statically indeterminate stress systems

Objectives:

- understand procedures for analysing statically indeterminate stress systems

- analyse various practical statically indeterminate stress systems

4.1 General solution procedures (p 82)

- equations of equilibrium of forces
- external forces from free body diagram
- internal force and stress relation
- equations of the geometry of deformation, or compatibility of displacements
- equations of load and deformation or stress and strain

4.2 Interaction between components of different stiffnesses (geometry or material) (pp 82-84)

4.3 Thermal problems (p 88-91)

Chapter 4 problems

Attempt 4.1, and also calculate the strain in each section and the total strain.
[Answer: ε_A = 586.5; ε_B = 162.5; ε_T = 375 $\mu\varepsilon$]

Attempt 4.3, 4.8 and 4.11 and if required 4.6.

Appendix A Properties of Areas

Objectives:

- Introduce/revise necessary concepts of centroid and second moment of area.
- Understand use of the Parallel Axis Theorem.

Appendix A

- Centroid (pp598-599)

$$\bar{y} = \frac{1}{A}\int_A y\,dA$$

$$\bar{z} = \frac{1}{A}\int_A z\,dA$$

- Second moment of area (pp 599-600)

About z-axis $I_z = \int_A y^2\,dA$

About y-axis $I_y = \int_A z^2\,dA$

- Parallel Axis Theorem (p 601)

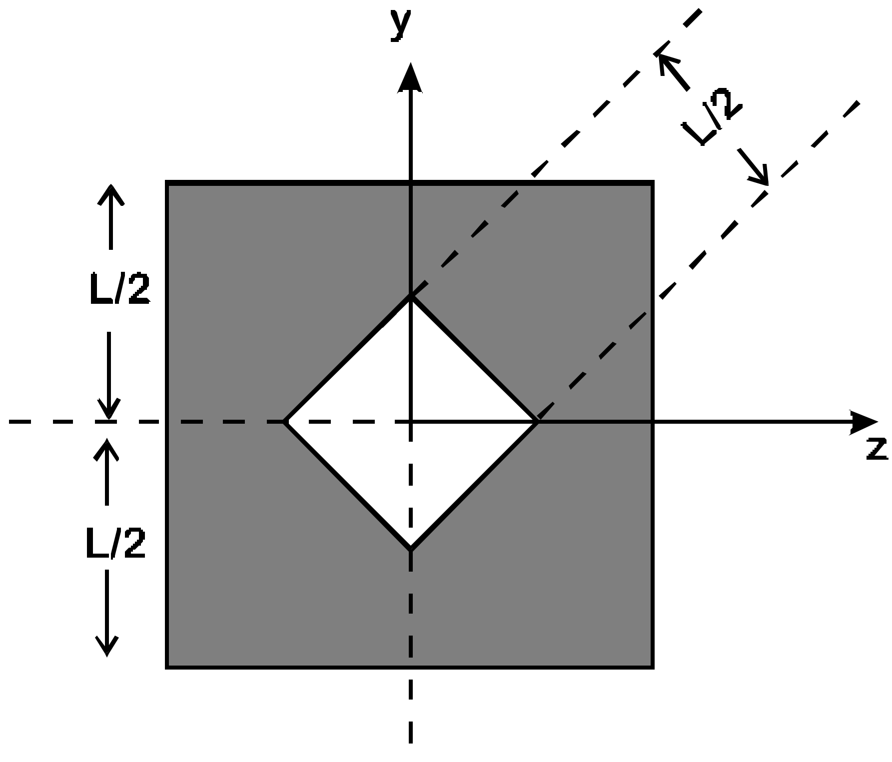
For z' axis parallel to z axis, and distance b from z axis,

$$I_{z'} = I_z + b^2 A$$

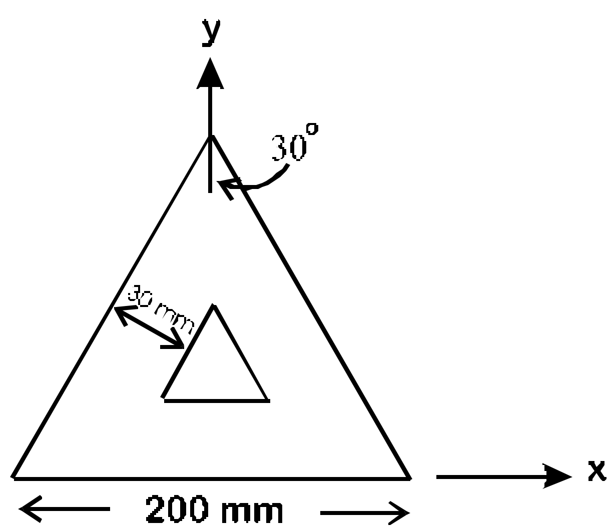
where A is the area of the shape.

Examples

1. The shaded area results from moving the central square from the outer square. Where is its centroid? Find the second moment of area about the z axis. Ans $I_z = 0.0781L^4$.



2. The area illustrated is formed by removing a small equilateral triangle from a large one. Find the location of the centroid G and the second moment of area about an axis through G parallel to the x axis.



Chapter 6 Bending Stress

Objectives:

- to be familiar with static equilibrium and determination of maximum bending moment (revision)
- to be familiar with determination of the centroid and second moments of area of beams of various cross-section
- to understand bending problems in the context of beam design
- to understand the derivation of the bending relationship
- to be able to use the principle of superposition
- to be able to use the bending relationship to solve problems

6.1 Revision of shear force and bending moment (pp 125-129)

6.2 Relationships between loading, shear force and bending moment (pp 130-131)

Differential forms:

- $$w = -\frac{dQ}{dx}$$

- $$Q = \frac{dM}{dx}$$

Integral forms

- $$Q_2 - Q_1 = \int_1^2 -w\,dx$$

- $$M_2 - M_1 = \int_1^2 Q\,dx$$

6.3 Stress in pure bending (pp 132-133)

6.4 Deformation in pure bending (pp 133-134)

- Longitudinal deformation

- Transverse deformation

6.5 Stress-strain relationship and normal stress in beams (pp 134-135)

- $\frac{\sigma_x}{y} = \frac{E}{R}$

6.6 Equilibrium of forces and moments (pp 135-136)

6.7 The bending relationship (p136)

- $\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$

6.11 Combined bending and end loading (pp 147-148)

- principle of superposition

6.12 Eccentric end loading (pp 148-149)

Chapter 6 Problems

Attempt 6.1, 6.2, 6.10 and 6.22

Chapter 7 Bending Slope and deflection

Objectives:

- understand importance of beam deflection in engineering design.
- understand deflection and slope equations.
- understand step function and Macaulay's method
- determination of slope and deflection using Macaulay's method.

7.1 Introduction (p 185)

- design beams for strength (Chapter 6)
- design beams for stiffness (chapter 7)

7.2 The curvature-bending moment relationship (pp 185-186)

7.3 Macaulay's Method (pp 193-202)

7.4 Procedures using Macaulay's method

- express BM in a single expression in terms of the step function
- integrate the slope and deflection equation
- determine the two boundary conditions
- solve the two integration constants from the two boundary conditions
- obtain the deflection and slope equations
- analyse the deflection/slope equations to find maximum deflection

Chapter 7 Problems

Attempt 7.2, 7.3

Chapter 8 Stress and strain transformations

Objectives:

- understand transformation of stresses in different axis sets.
- to be able to understand and use concepts of principal stresses.
- to be able to derive maximum shear stress for any stress field.

8.1 Introduction (p 292)

- one-dimensional stress systems - normal stress.
- two-dimensional stress systems - normal and shear stress.

8.2 Derivation of transformation equations (pp 293-296)

- normal stress on oblique plane.
- shear stress on oblique plane.

8.3 Mohr's stress circle (pp 298-300)

8.4 Principal stresses and maximum shear stress (pp 301-303)

Chapter 8 Problems

11.7, 11.19, 11.21