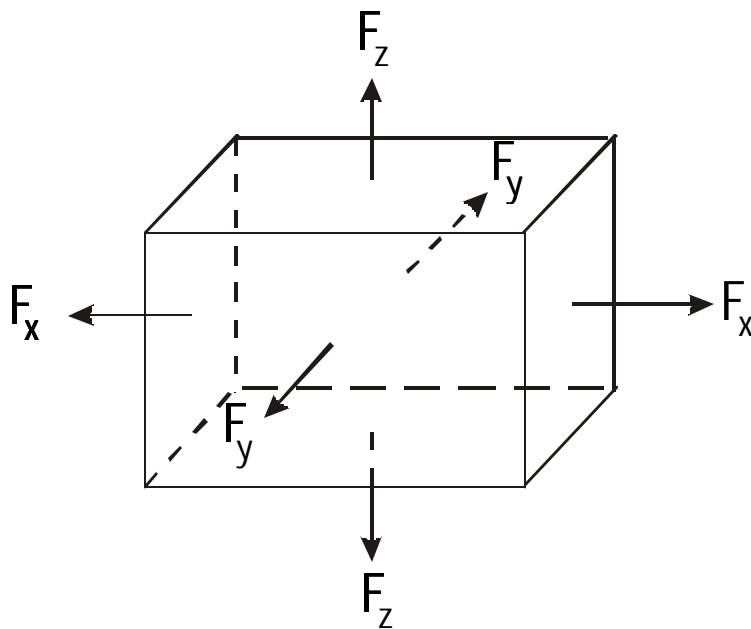


EQUILIBRIUM

EQUILIBRIUM OF FORCES

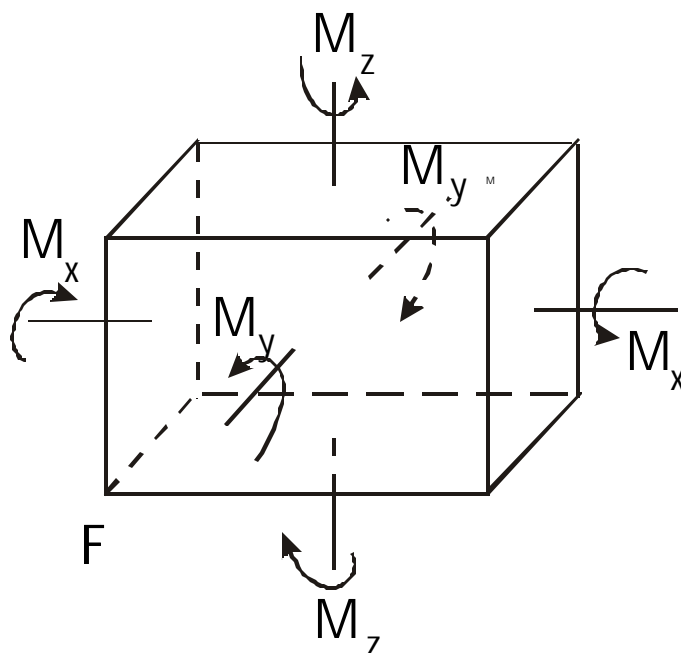


$$\Sigma F_x = 0$$

$$\Sigma F_y = 0$$

$$\Sigma F_z = 0$$

EQUILIBRIUM OF MOMENTS

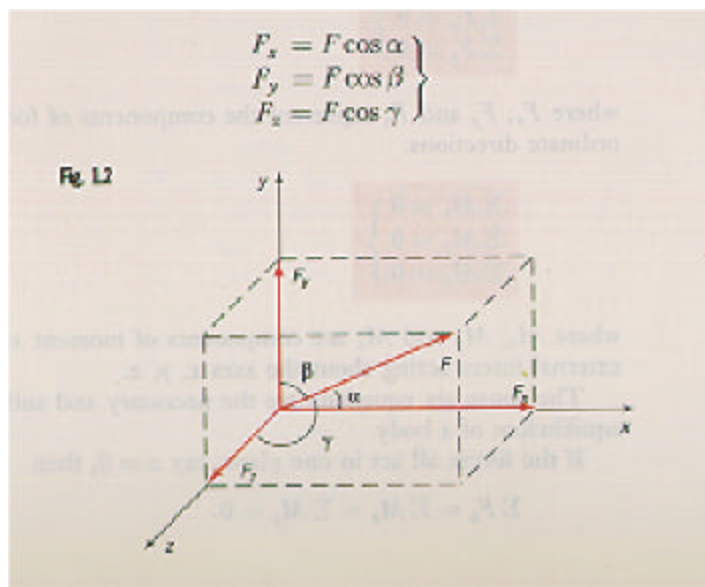
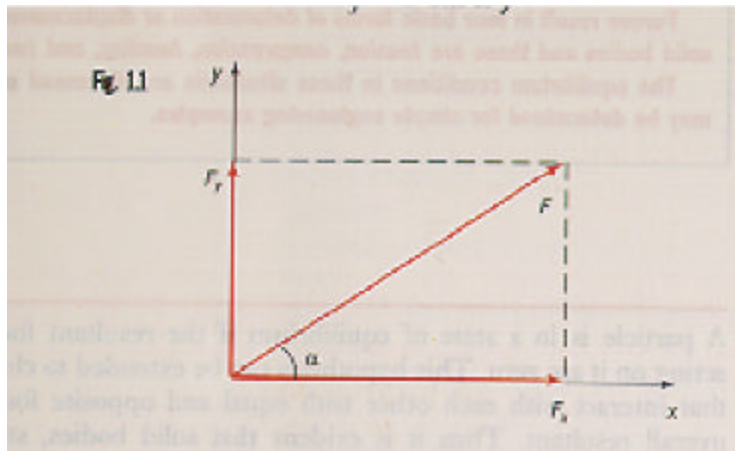


$$\Sigma M_x = 0$$

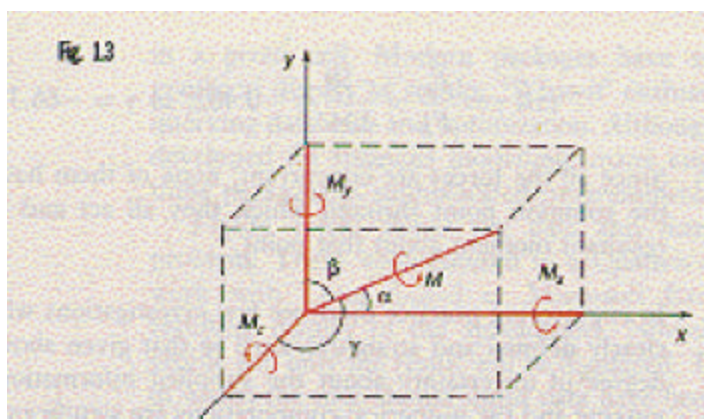
$$\Sigma M_y = 0$$

RESOLVE FORCES AND MOMENTS – TEXTBOOK DIAGRAMMS

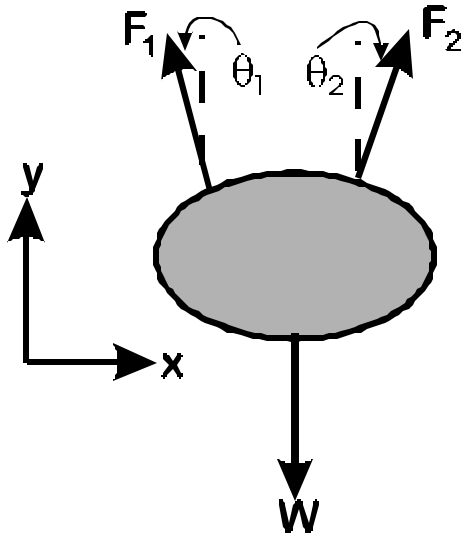
Resolution of Forces



Resolution of moments



EXAMPLE – EQUILIBRIUM AND RESOLUTION OF FORCES



Choose directions

Resolve along y: $F_1 \cos \theta_1 + F_2 \cos \theta_2 - W = 0$ (1)

Resolve along x: $F_2 \sin \theta_2 - F_1 \sin \theta_1 = 0$ (2)

From (2), $F_2 = \frac{F_1 \sin \theta_1}{\sin \theta_2}$

Use in (1):

$$F_1 (\cos \theta_1 + \sin \theta_1 \cos \theta_2 / \sin \theta_2) = W$$

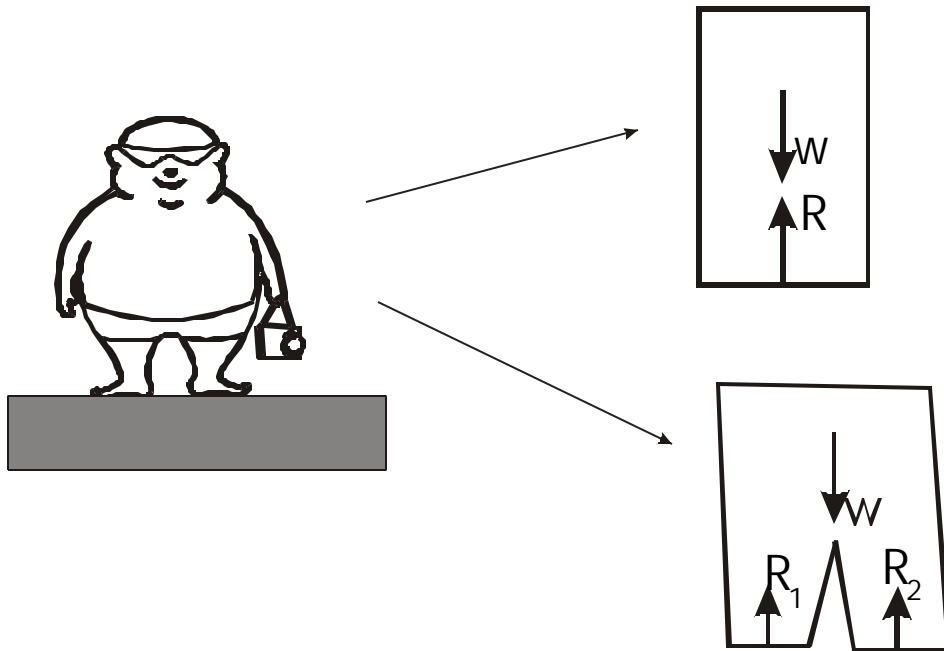
Rearrange:

$$F_1 = W / (\cos \theta_1 + \sin \theta_1 / \tan \theta_2)$$

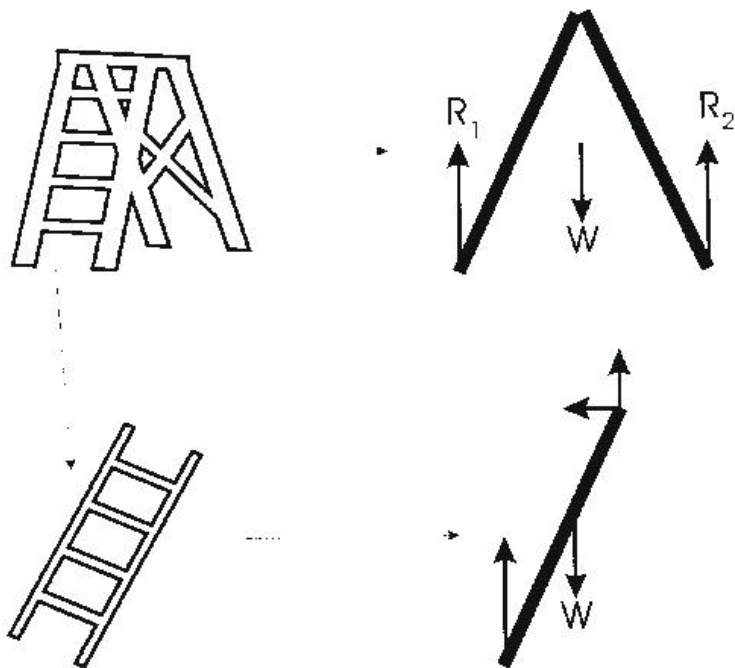
Similarly

$$F_2 = W / (\cos \theta_2 + \sin \theta_2 / \tan \theta_1)$$

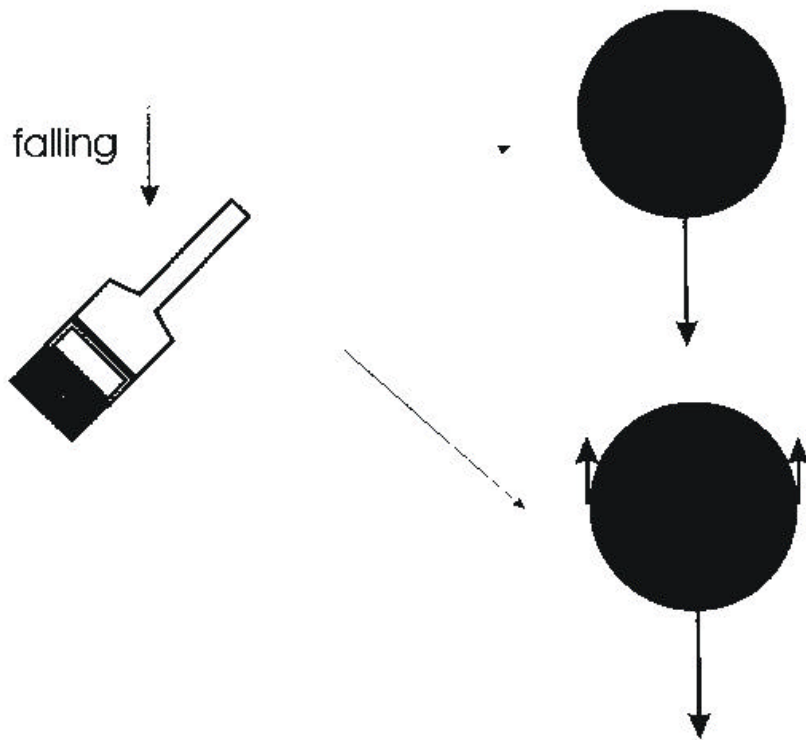
FREE BODY DIAGRAMS



FREE BODY DIAGRAMS



FREE BODY DIAGRAM - NON-EQUILIBRIUM

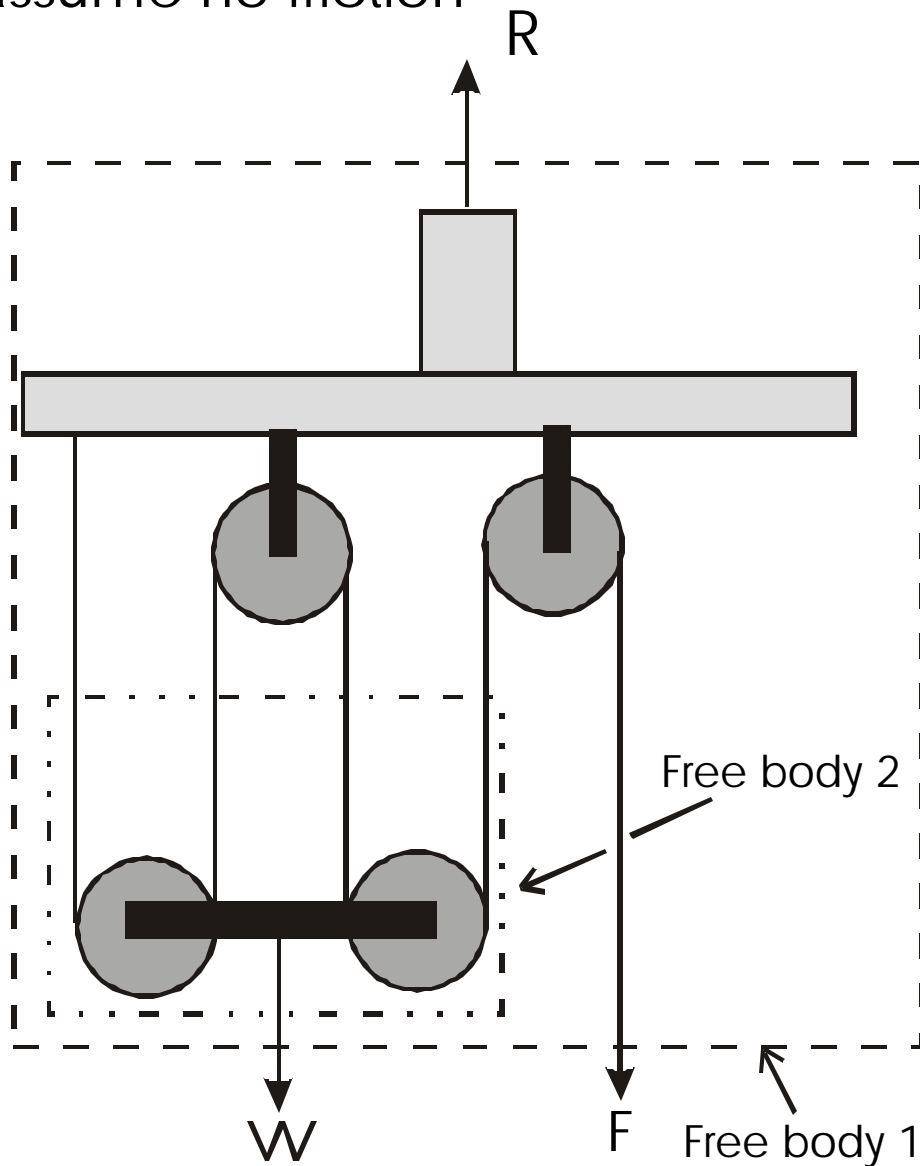


Example

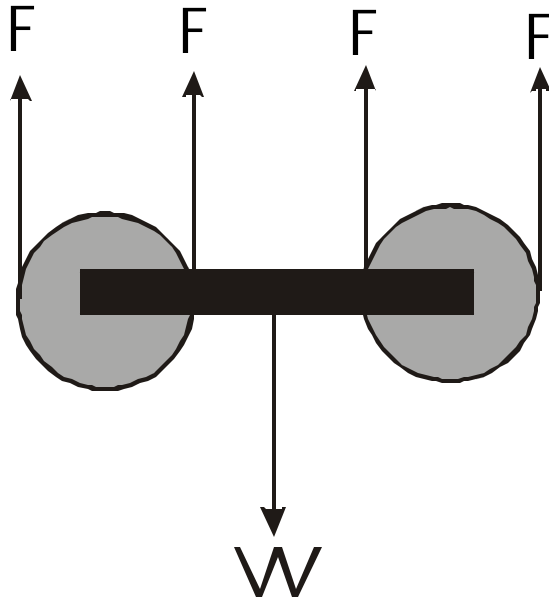
What is the force R in terms of F and W ?
(free body 1)

What is F in terms of W ? (free body 2)

-assume no friction



Body 2



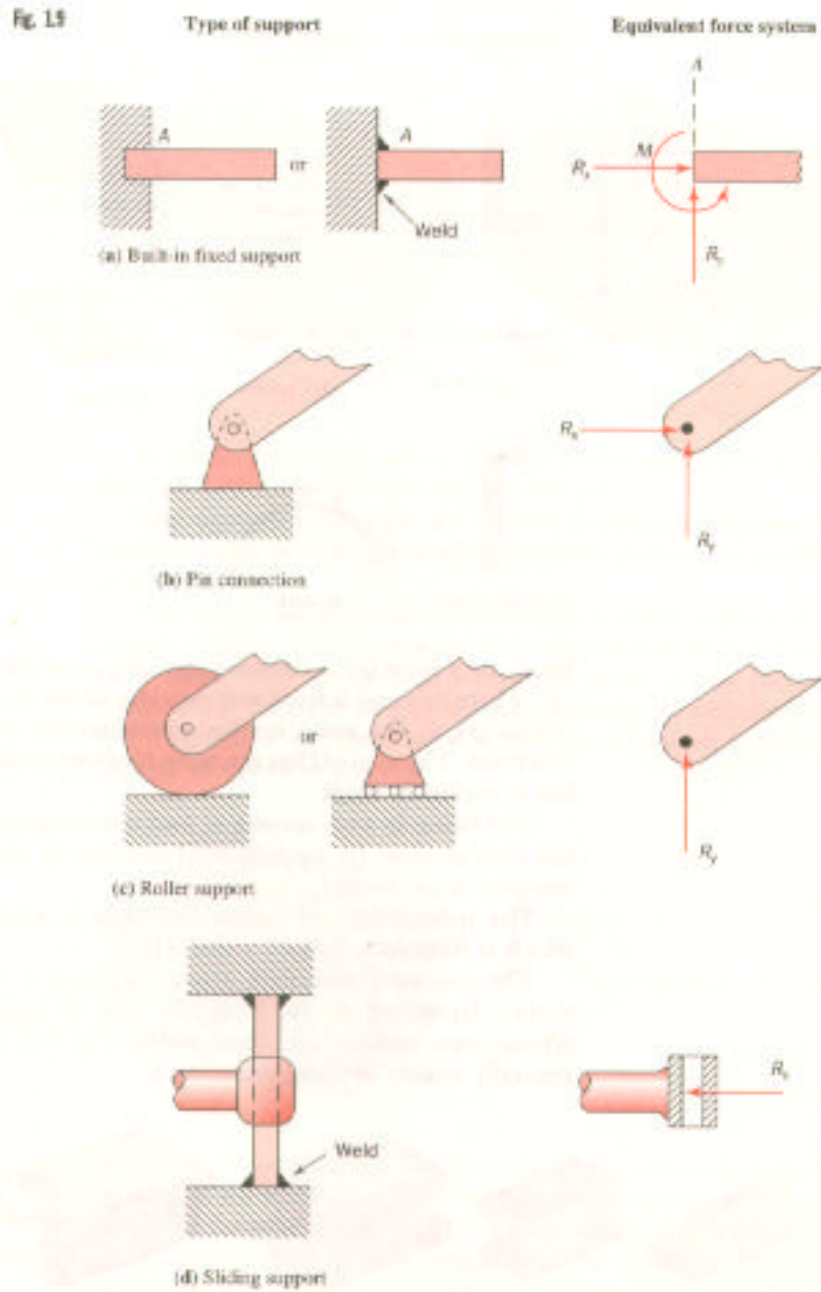
Vertical forces:

$$4F = W$$

i.e. $F = W/4$

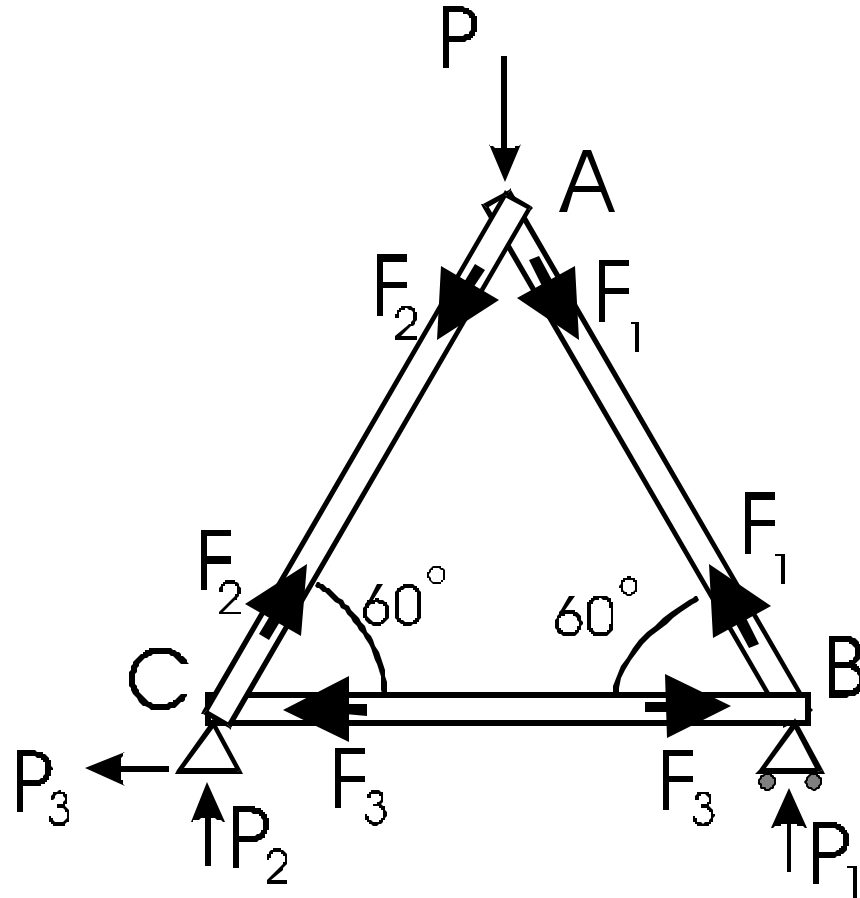
DETERMINACY

Types of Support

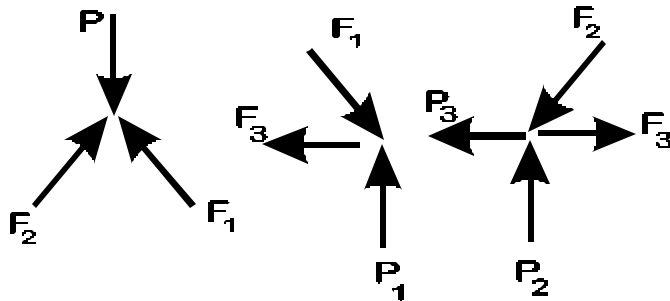


	Displacement			Reacting force		
	x	y	θ	R_x	R_y	M_o
(a) Built-in or fixed support				✓	✓	✓
(b) Pin connection			✓	✓	✓	
(c) Roller support	✓		✓		✓	
(d) Sliding support		✓		✓		✓

Determinacy - pin-jointed frame example



Joint A Joint B Joint C



A: horizontal $F_1 \cos 60^\circ - F_2 \cos 60^\circ = 0$: $F_1 = F_2$ (1)

vertical $P - F_1 \cos 30^\circ - F_2 \cos 30^\circ = 0$ (2)

B: horizontal $F_3 - F_1 \cos 60^\circ = 0$ (3)

vertical $P_1 - F_1 \sin 60^\circ = 0$ (4)

C: horizontal $F_3 - P_3 - F_2 \cos 60^\circ = 0$ (5)

vertical $P_2 - F_2 \sin 60^\circ = 0$ (6)

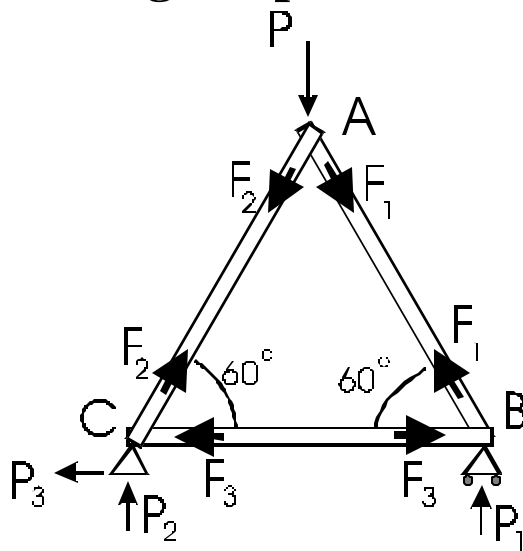
Important equation for two-dimensional cases

$$m + r = 2j$$

where m is the number of forces in members; r the number of reactions; and j the number of joints. In the above example, $m=3$, $r=3$ (2 vertical, 1 horizontal) and $j=3$. Equation balances. Note left hand side is the number of unknowns, and right hand side the number of equations. When these are equal, the problem can be solved, as can the one above. Such a problem is termed *just stiff*, or *statically determinate*.

Determinacy - pin-jointed frames

Original problem



Unknowns $F_1, F_2, F_3,$

$P_1, P_2, P_3.$

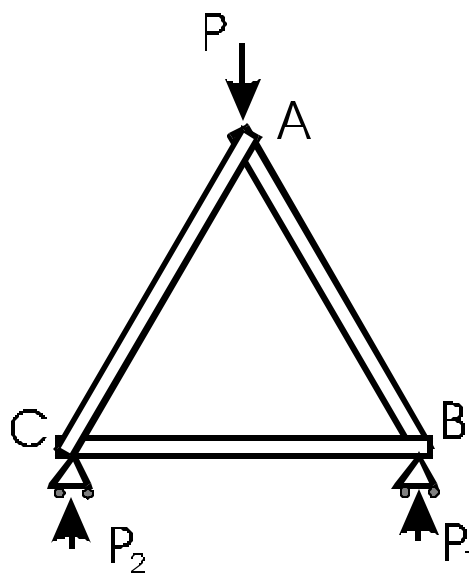
6 equations

$$m + r = 2j$$

$$(3 + 3 = 2 \cdot 3)$$

just stiff

rollers added



Unknowns $F_1, F_2, F_3,$

$P_1, P_2.$

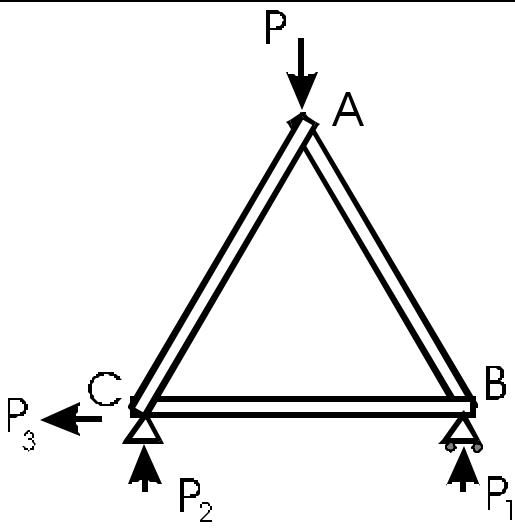
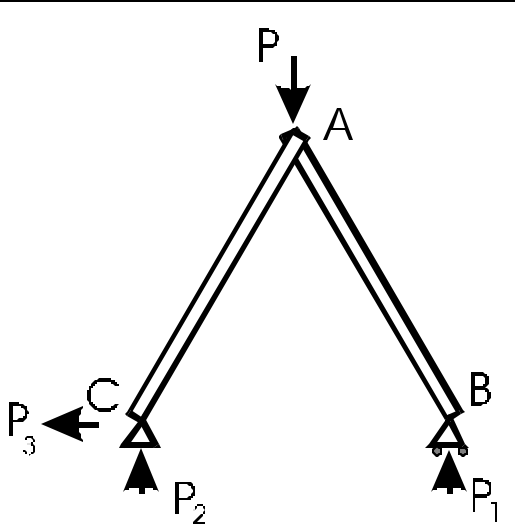
6 equations

$$m + r < 2j$$

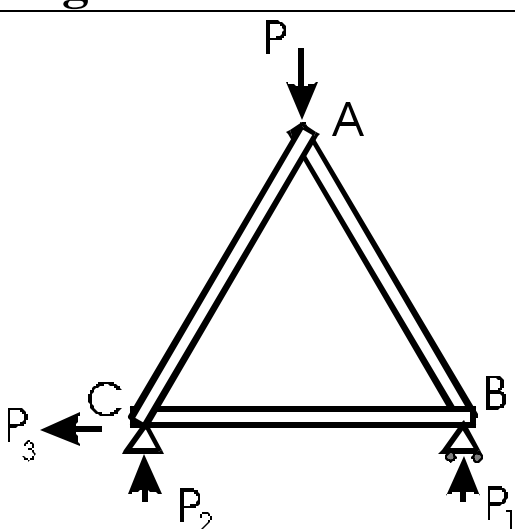
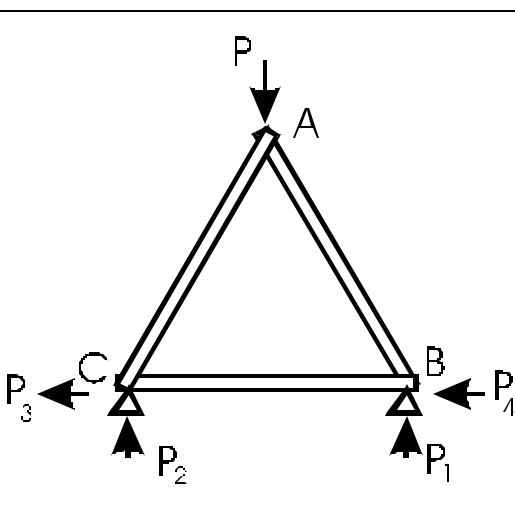
$$(3 + 2 < 2 \cdot 3)$$

under-stiff

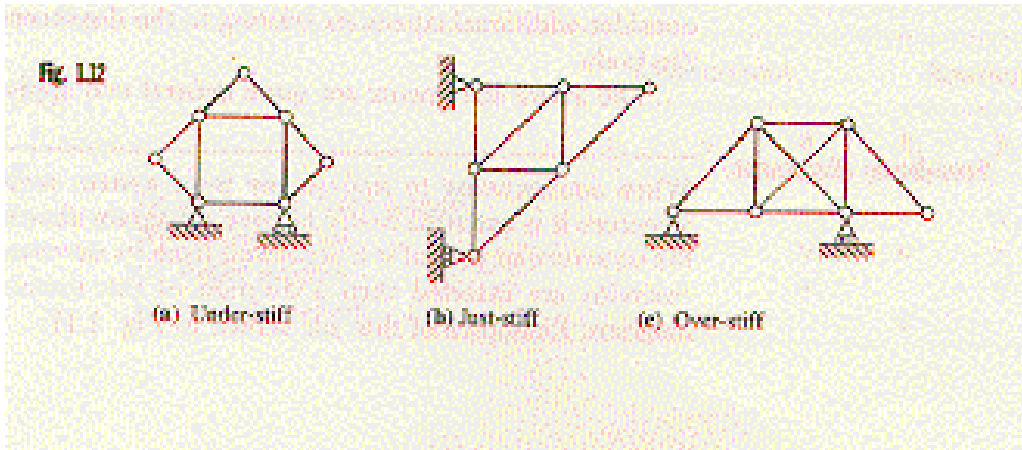
Under-stiff scissors

original structure	scissor mechanism
	
$m + r = 2j$ $(3 + 3 = 2 \cdot 3)$	$m + r < 2j$ $(2 + 3 < 2 \cdot 3)$

Over-stiff structure

original structure	rollers removed
	
$m + r = 2j$ $(3 + 3 = 2 \cdot 3)$	$m + r > 2j$ $(3 + 4 > 2 \cdot 3)$

Textbook examples



Determinacy of beams

Beams - types of loading and support

point load



distributed load



simple support

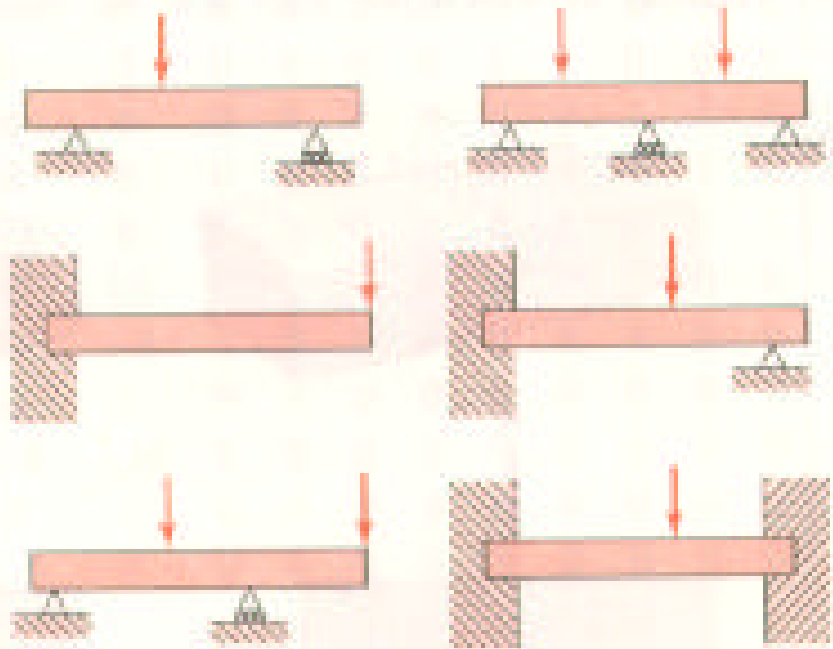


built-in end



Textbook examples

Fig. 1.27

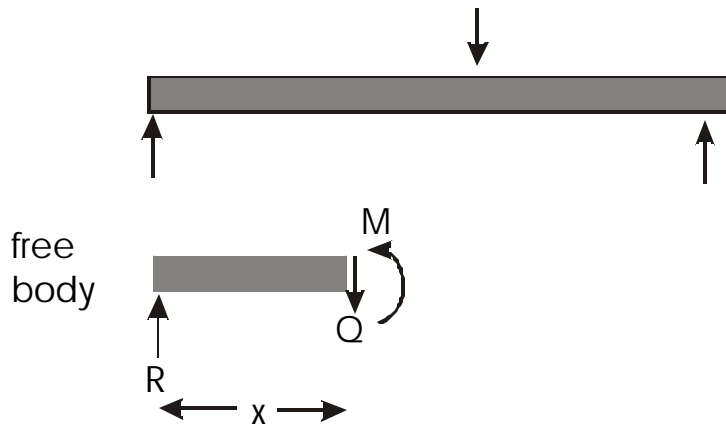


(a) Statically determinate

(b) Statically indeterminate

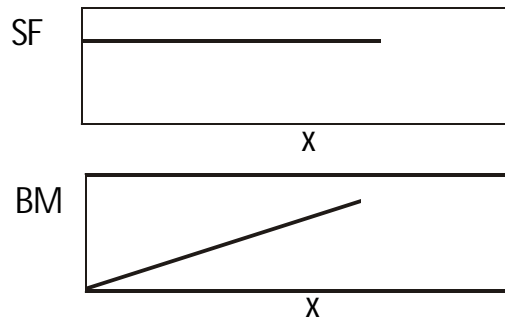
SHEAR FORCE AND BENDING MOMENT

We can subdivide a free body in order to analyse it (pulley example). This also applies to beams, and leads to the concepts of shear force and bending moment.



Shear force $Q = R$

Bending moment $M = Rx$



Example - cantilever

