Structural optimization in occupant safety and crash analysis

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Abstract In occupant safety and crash analysis, non-linear structural finite element codes based on explicit integration schemes are used. Parameter changes are derived from the results of the analysis to improve the design in order to meet the requirements of regulations and design goals. Today, the common approach is to use a trial-and-error search. This is very expensive with respect to labor costs. Optimization methods can be used to automate the search for the optimum design. Closed loop software for the optimization of structures with non-linear behavior such as is known for linear static analysis is not available currently. Usually add-ons to the non-linear codes based on response surface methods or finite difference sensitivity analysis are used. The paper discusses applications of optimization tools in occupant safety and crash analysis using a very efficient response surface method. Some examples are given.

Introduction

In automotive design, the occupant and structural behavior in the event of a crash is of special interest. Non-linear finite element and rigid body analysis is applied to predict the responses of the structure or of the occupant. Decisions based on these computations can lead to significant design modifications. Usually, intuition leads the iterative process of finding the best design. It is often hard to determine necessary design modifications from the analysis results only. In some cases many variations are tried before a satisfactory design is found.

Structural optimization, based on computational methods, is a useful tool to support the process of finding the right design. The design layout can be described mathematically as an optimization problem. Using the results of a computational optimization, the decision process can be improved. Optimization of structural elements can lead to significant improvements of the final design.

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The optimization problem appears as

\[
\begin{align*}
\text{Objective :} & \quad W(\mathbf{p}) \Rightarrow \min \\
\text{Subject to} & \quad \mathbf{M}^T \ddot{\mathbf{d}} = {}^T f(\dot{\mathbf{d}}, \ddot{\mathbf{d}}, \mathbf{p}), \ 0 \leq t \leq t_c \\
& \quad \text{with} \quad 0\mathbf{d} = \mathbf{d}_0, 0\dot{\mathbf{d}} = \dot{\mathbf{d}}_0 \\
\text{Constraints :} & \quad g(\mathbf{d}, \dot{\mathbf{d}}, \ddot{\mathbf{d}}, \mathbf{p}) \leq 0 \\
\text{Side constraints :} & \quad \mathbf{p}^l \leq \mathbf{p} \leq \mathbf{p}^u
\end{align*}
\]

Optimization of the structure can be performed to obtain a certain occupant or structural response or to reduce the structural mass as much as possible. The objective function \(W\) can be any structural response, such as the total or a weighted mass of the structure. The constraint functions \(g = \{g_i\}, i = 1, \ldots, n_c\) are occupant or structural responses. The state equation describes the dynamic behavior of the structure that is to be optimized. Here, it is given in the form of an explicit integration scheme, such as is usually applied in occupant and crash analysis. The matrix \(\mathbf{M}\) is the time independent mass matrix. The state vector \(\dot{\mathbf{d}}\) contains the generalized displacements. The load vector \(\mathbf{f}\) contains external and internal forces. The quantity \(\mathbf{p} = \{p_j\}, j = 1, \ldots, m\) is the vector of design variables.

Industrial application of structural optimization often depends on the availability of software products. For linear static and dynamic problems such software is available and fairly well supported. Layout and shape optimization can be performed to design structural parts. In the design process based on crashworthiness the trial and error approach is still the most commonly used method to find a design that fits the needs. For analysis, explicit integration schemes are mostly used for structural analysis. No commercial software is available to perform sensitivity analysis and optimization for crashworthiness problems. A review of the optimization of structures under impact loading can be found in Schramm and Pilkey (1996).

**Design process**

Viewing the design process as an optimization process to find structures and structural parts that fulfill certain expectations toward their economy, functionality and appearance, structural optimization can be seen as the driver in this process. It does not matter what functionality is expected from the design.

Figure 1 shows the design process in form of an optimization iteration going through the following steps:

1. Define an initial design \(\mathbf{p}^{(s=0)}\);
2. Analyze the behavior of the structure;
3. Compare the results of the analysis with the requirements, such as allowable stresses, displacements, cost limits, injury criteria or energies;
(4) If the requirements are not met, change the design such that \( p^{(s+1)} = p^{(2)} + \delta p \); 

(5) Back to (2).

It is possible to determine the search direction \( \delta p \) directly from the results of the structural analysis. Then, the general formulation of the optimization problem (1) appears as:

\[
\begin{align*}
\text{Objective} & : & \psi_0(p) \Rightarrow \text{min} \\
\text{Constraints} & : & \psi_i(p) \leq 0 \\
\text{Design space} & : & p^l \leq p \leq p^u
\end{align*}
\]

Objective and constraints \( \psi_i(p), i = 0, \ldots, n_c \) can be approximated for each design \( p^{(s)} \) using the series expansion

\[
\psi_i = \psi_i(p^{(s)}) + \sum_{j=1}^{m} \frac{d\psi_i}{dp_j} \delta p_j
\]

The gradient \( d\psi_i/dp_j \) can be obtained directly from the results of the numerical analysis. If the gradient is known, the search direction \( \delta p \) can be obtained from the solution of an approximate optimization problem.

**Optimization methods**

For the solution of structural optimization problems, approximation methods are well established. Similar to the description in the previous section, the sequential solution of approximate optimization problems leads to the solution of the optimization problem (1). Also in conjunction with crash analysis these optimization methods are applicable. In general two types of approximation methods can be distinguished. First, the local approximation using gradient information, and second, global approximation methods that use an approximation to the original structural optimization problem over a wide range of the design variables (Barthelemy and Haftka, 1993).
Local approximation

Local approximation methods determine the solution of the optimization problem using the following steps:

1. Analysis of the physical problem using finite elements;
2. Convergence test, if the solution is found;
3. Design sensitivity analysis;
4. Solution of an approximate optimization problem formulated using the sensitivity information;
5. Back to (1).

This approach is based on the assumption that only small changes of the design occur in each optimization step. The result is a local minimum. It should be mentioned that the biggest changes occur in the first few optimization steps. Therefore, not too many system analyses are necessary in practical applications.

The design sensitivity analysis produces the gradient of the structural responses with respect to the design variables. It is one of the most important ingredients in local approximation methods.

Regarding implementation effort, the simplest method for design sensitivity analysis is to use finite differences. In the form of central differences the design sensitivities for a function $\psi_i$ appear as

$$\frac{d\psi_i}{dp_j} = \frac{\psi_i(p_j + \Delta p_j) - \psi_i(p_j - \Delta p_j)}{2\Delta p_j}$$

The numerical effort for optimization using this method is quite large. Optimization for crash codes using finite difference sensitivity analysis is already available.

Much more efficient with regard to the numerical effort would be an analytical sensitivity analysis. In the case of using an explicit integration scheme for the solution of the structural problem this sensitivity analysis can be derived from the state equation. The state equation is usually solved using central differences for the time domain.

Then, assuming a constant time step $\Delta t$,

$$t\ddot{d} = M^{-1} (t'f)$$

$$t+\Delta t/2 \ddot{d} = t-\Delta t/2 \ddot{d} + t\dddot{d}\Delta t$$

$$t+\Delta t \dot{d} = t \dot{d} + t+\Delta t/2 \ddot{d}\Delta t$$

This notation follows that of Hallquist (1997).

Differentiation of these equations with respect to the design variables $p_j$ yields
Using the derivatives of the state vector, the gradient of the objective and constraint functions can be computed.

An adjoint variable method for sensitivity analysis and more detail can be found in Schramm and Pilkey (1996).

**Global approximation**

The implementation of a design sensitivity analysis for explicit integration schemes is not a simple task. Analytical sensitivity analysis is not yet commercially available. Therefore, it is common to introduce higher order analytical expressions to approximate the dependence between the objective or constraint functions and the design variables. An approximate analytical relationship between structural responses and design variables can be established with only a few analyses.

Let $\psi_i(p)$ be the response of interest. Then, a polynomial $\hat{\psi}_i(p)$, called response surface, of the degree $n$ can be introduced such that

$$
\psi_i(p) \approx \hat{\psi}_i(p) = a_{i0} + \sum_{j=1}^{m} a_{ij}p_j + \sum_{j=1}^{m} \sum_{k=1}^{m} a_{ijk}p_jp_k + \ldots
$$

The quantities $a_{i0}, a_{ij}, a_{ijk}$ are constants that are determined from the results of the system analyses by solving a linear system of equations. The number of analyses that are necessary depends on the polynomial degree $n$ (1). If the mixed terms are omitted, the computational effort can be reduced considerably. Then, the response surface appears as

$$
\psi_i(p) \approx \hat{\psi}_i(p) = \sum_{l=0}^{n} \sum_{j=1}^{m} a_{lj}p_j^l
$$

Using the equations above, an approximation to the optimization problem can be established. It appears as

$$
\hat{\psi}_0(p) \Rightarrow \min
\hat{\psi}_i(p) \leq 0, i = 1, \ldots, n_c
$$

The solution to this problem can be determined using mathematical programming. It yields an approximate solution to the structural optimization problem (1).
The solution of the optimization problem (1) using response surfaces involves the following steps:

1. Finite element solutions of the problem;
2. Response surface computation for each response;

Problems of different physical content can be combined in one optimization problem easily.

Using the method above, a predefined number of designs is analyzed followed by the computation of the response surfaces and the optimization solution. The general idea of this method is shown in Figure 2 for a problem with one design variable.

A different approach is to analyze the designs as the optimization proceeds. This method will be called sequential response surface method (Altair HyperOpt, User’s Manual, 1997) (see Figure 3). Take a quadratic response surface

\[ \psi_i(p) \approx \hat{\psi}_i(p) = a_{i0} + \sum_{j=1}^{m} a_{ij}p_j + \sum_{j=1}^{m} \sum_{k=1}^{m} a_{ijk}p_jp_k \] (10)
The coefficients \(a_{i0}, a_{ij}, a_{ijk}\) are determined using a least square fit of the function \(\hat{v}_i(p)\) on the existing designs. In the first step a number of designs \(s = 1 + m\) is analyzed and the coefficients \(a_{i0}, a_{ij}\) are determined. The next \(m\) designs are analyzed to determine the coefficients \(a_{ijj}\) which are the diagonals of the Hessian. Additional designs are used to determine the remaining coefficients, as it becomes necessary. After \(s = 1 + m + (1 + m)m/2\) designs, an algorithm is used to weigh the designs.

The optimization procedure is as follows:

1. Analyze the initial design and \(m\) perturbed designs;
2. Use a least square fit to determine the polynomial coefficients for the objective function \(\hat{v}_0(p)\) and each constraint \(\hat{v}_i(p)\):
   - If \(s = 1 + m\), then calculate \(a_{i0}, a_{ij}\);
   - If \(1 + m < s < 2(1 + m)\), then compute \(a_{i0}, a_{ij}, a_{ijk}, j = 1, s - m - 1\);
   - If \(1 + 2m < s < 2 + m + (1 + m)m/2\), then calculate \(a_{i0}, a_{ij}, a_{ijk}\) and the successive off-diagonal elements \(a_{ijk}\);
   - If \(s > 2 + m + (1 + m)m/2\), then weigh the designs and calculate all \(a_{i0}, a_{ij}, a_{ijk}\);
3. Solve for the approximate optimum design using mathematical programming;
4. Analyze the approximate optimum design;
5. If the designs have converged, stop;

**Applications**

**Optimization of a rail**

The design of a rail hit by a stone wall is used as the first example. Figure 4 shows the deformed rail. The rail is clamped at the right end and loaded with a stone wall of a mass of 1,000kg from the left side. The stone wall has an initial velocity of 2m/s. The length of the rail is \(l_x = 800\) mm.

The system needs to be designed such that it absorbs a maximum of energy during the computation time of \(t_e = 0.03\) s. The displacement of the stone wall in \(x\) direction should be \(\delta l_x = 50\) mm.

Problems of this kind are common in the case of the design of car bumpers and crash boxes. In many cases it is not possible to determine the optimum design using the trial and error approach. Optimization is the best approach to get a solution.

A sizing optimization has been performed using the thickness of six shell patches as design variables (DV\(i\), Figure 5). The initial thickness of each element was 2mm and could be modified from 0.2mm to 3mm.

The structural analysis to obtain the displacement and energy absorbed was performed using LS-Dyna (Hallquist, 1997). Table I summarizes the results. The code DoE (1997) provided the ordinary response surface solution using a
Figure 4.
Rail: deformed initial design

Figure 5.
Rail: sizing optimization; design variables

<table>
<thead>
<tr>
<th></th>
<th>Initial design</th>
<th>Ordinary RS</th>
<th>Sequential RS</th>
<th>Local approximation</th>
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</thead>
<tbody>
<tr>
<td>DV1 [mm]</td>
<td>2.00</td>
<td>1.70</td>
<td>2.02</td>
<td>2.25</td>
</tr>
<tr>
<td>DV2 [mm]</td>
<td>2.00</td>
<td>1.92</td>
<td>1.57</td>
<td>2.95</td>
</tr>
<tr>
<td>DV3 [mm]</td>
<td>2.00</td>
<td>0.20</td>
<td>1.56</td>
<td>2.11</td>
</tr>
<tr>
<td>DV4 [mm]</td>
<td>2.00</td>
<td>1.95</td>
<td>1.65</td>
<td>1.94</td>
</tr>
<tr>
<td>DV5 [mm]</td>
<td>2.00</td>
<td>2.63</td>
<td>3.00</td>
<td>2.28</td>
</tr>
<tr>
<td>DV6 [mm]</td>
<td>2.00</td>
<td>3.00</td>
<td>2.48</td>
<td>2.28</td>
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<tr>
<td>$E_{\text{max}}$ [Nm]</td>
<td>1010</td>
<td>1177</td>
<td>1254</td>
<td>1170</td>
</tr>
<tr>
<td>$\Delta l$ [mm]</td>
<td>52.8</td>
<td>49.95</td>
<td>49.88</td>
<td>49.85</td>
</tr>
</tbody>
</table>

Table I.
Rail: sizing optimization results
cubic response surface as (1). Altair HyperOpt (1997) was applied to obtain the optimization solution using the sequential response surface method. Further, another optimum design using a local approximation approach with finite difference sensitivity analysis has been obtained.

All three methods did yield approximately the same results for the structural behavior. The thickness distribution is somewhat different. This shows that different solutions to the problem exist.

In another investigation the same rail was designed using shape optimization. The optimization problem was the same as stated above. Again six design variables are used. Figure 6 shows the iteration history, and Figure 7 shows the initial and final shape. One can see that the target value of the displacement could be reached within a certain bound and the strain energy could be maximized.

**Airbag**

In another application, an airbag impactor test was simulated. The goal was to find a parameter combination that would minimize the Head Injury Criterion (HIC), which is an occupant injury criterion that provides a measurement of the load on the occupant’s head. The HIC is computed from the deceleration of the impactor such that

\[
\text{HIC} = \max_{t_2 - t_1 \leq 36 \text{ms}} \left( \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \dot{a}_{\text{Impactor}} dt \right)^{2.5} (t_2 - t_1)
\]  

Its value should never exceed 1,000. The constraint was that the impactor should not move more than 260mm. The design variables are typical for airbag design such as the vent orifice area, the permeability of the fabric and the ignition time. Also, the mass flow of the gas has been modified using five points on the mass flow time curve as design variables. Figure 8 shows the test setup. The iteration history and the structural behavior is shown in Figures 9
and 10, respectively. The result was a parameter combination that reduces the HIC considerably by letting the displacement be far below the limit.

The dynamic analysis was performed using PAM-CRASH (1997). The optimum design has been obtained using the sequential response surface method implemented in *Altair HyperOpt* (1997).

**Roof crush layout**
In automotive design certain government regulations have to be fulfilled by the design. Numerical analysis is used to check if a design complies with these
Design Optimization

Figure 9.
Airbag: iteration history

Figure 10.
Airbag: responses, initial and final design

rules to avoid expensive testing. Here the US standard FMVSS216 is of interest. This standard requires a test that simulates a roll-over of the car. The goal is that the A-pillar withstands a certain load. Figure 11 shows the test as it is given by the standard.

Figure 11.
Roof crush test
FMVSS216: test setup
In the test setup a plate is pushed on the A-pillar with constant velocity. The load-displacement curve is measured. In a simulation the test setup is modeled using finite elements. Just the region of interest is modeled and not the whole structure.

The objective in this optimization is to minimize the mass of the design and still hold the required load level. The load-displacement curve was discretized in five points to define the constraints. The curves for the initial and final design are shown in Figure 12. Design variables have been the shell thickness of five material properties in the A-pillar and the roof frame.

It can be seen that the initial design was well below the bound for most of the simulation time. The optimization did yield a design that would exceed the load limit. Usually a large number of design variations is used before a result is obtained. For the example problem 13 designs have been tried automatically.

The analysis has been performed using LS-Dyna (Hallquist, 1997) and the optimization using Altair HyperOpt (1997).

**Knee bolster design**

In this example, rigid body simulation is used to analyze the occupant behavior and to make initial package studies for a knee bolster. The use of optimization will be demonstrated (Schramm et al., 1997). For the investigation, just the lower extremities of the dummy are modeled (Figure 13). The dummy is loaded with an acceleration pulse $a_x(t)$ resulting from a crash test according to the US government regulation FMVSS208. The model of the initial design was validated against a sled test.

The objective of the optimization is to minimize the femur load with a given limit on the knee penetration into the knee bolster. The optimization problem appears as

$$F_{Femur}(\Delta \theta, \Delta k_x, \Delta l_x, \Delta l_z, \Delta \alpha, \ldots) \Rightarrow \min$$

$$\delta(\Delta \theta, \Delta k_x, \Delta l_x, \Delta l_z, \Delta \alpha, \ldots) \leq 0.07m$$

(12)

![Figure 12. Roof crush test: load-displacement (time) curves for initial and final design](image)
The design variables are defined in Figure 14. The optimization was performed using the sequential response surface method. The results are summarized in Table II.

### Figure 13.
Knee bolster design model

### Figure 14.
Knee bolster design: design variables

<table>
<thead>
<tr>
<th>FV</th>
<th>Lower bound</th>
<th>Upper bound</th>
<th>Optimal value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta \theta[^\circ] )</td>
<td>-10.00</td>
<td>10.00</td>
<td>-10.00</td>
</tr>
<tr>
<td>( \Delta k_1[\text{mm}] )</td>
<td>-0.10</td>
<td>0.07</td>
<td>-0.10</td>
</tr>
<tr>
<td>( \Delta l_1[\text{mm}] )</td>
<td>-0.05</td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>( \Delta l_2[\text{mm}] )</td>
<td>-0.05</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>( \Delta \alpha[^\circ] )</td>
<td>-5.00</td>
<td>5.00</td>
<td>-4.65</td>
</tr>
<tr>
<td>( F_{\text{Femur}}[\text{N}] )</td>
<td>Minimize</td>
<td>5332</td>
<td></td>
</tr>
<tr>
<td>( \delta[\text{mm}] )</td>
<td>0.07</td>
<td>0.0617</td>
<td></td>
</tr>
</tbody>
</table>

Table II.
Knee bolster design: results
The analysis has been performed using MADYMO (1997) and the optimization using Altair HyperOpt (1997).

**Concluding remarks**

The results from computational analyses can be employed effectively for the design of structural systems and parts if structural optimization technology is used. Such methods reduce the effort for expensive design variations and comparisons considerably. An objective oriented search for the best design is possible if the design problem is formulated as a structural optimization problem. Of course, such a formulation requires the accurate analysis of the physics of the problem and the sufficient determination of the input data such as loading, boundary conditions, material data, and so forth.

Today, the application of structural optimization is not just limited to linear problems, but it can also be applied to complex physical behavior, such as analyzed in crash and occupant simulations. Therefore such methods are of growing interest in the automotive industry. The examples in this paper show that structural optimization is a valuable tool in structural design even of structures with complex nonlinear dynamic behavior.

**References**


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